

Angle Link Lab

Use diagrams to prove angle and distance relationships.

HIGH SCHOOL • MA.912.GR.1.1

MA.912.GR.1.1 "Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles."

NAME _____

DATE _____

GIVE A REASON FOR EACH STEP

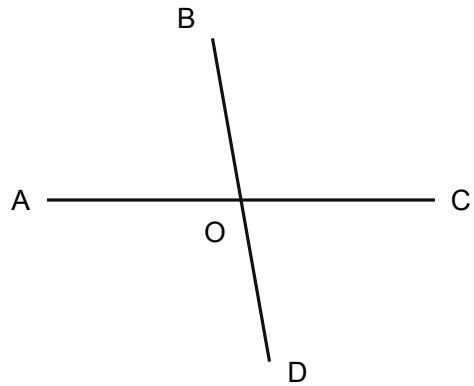
A linear pair sums to 180 degrees. Vertical angles are congruent because they supplement the same angle. Parallel lines give congruent corresponding and alternate interior/exterior angles, and supplementary consecutive interior angles. A point on a segment's perpendicular bisector is equidistant from its endpoints; the converse also holds. Use givens and marked relationships, not appearance or measurement. Write short proofs with a reason for every step.

WORKED EXAMPLE

Lines AC and BD intersect at O. Prove angle AOB is congruent to angle COD without assuming the vertical-angle theorem.

- $m(\angle AOB) + m(\angle BOC) = 180$ and $m(\angle BOC) + m(\angle COD) = 180$: each is a linear pair.
- The sums are equal. Subtract $m(\angle BOC)$ from both sides to get $m(\angle AOB) = m(\angle COD)$.

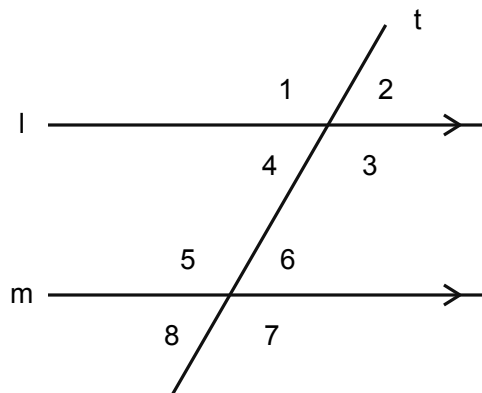
Answer: Equal angle measures imply congruent angles: AOB is congruent to COD.



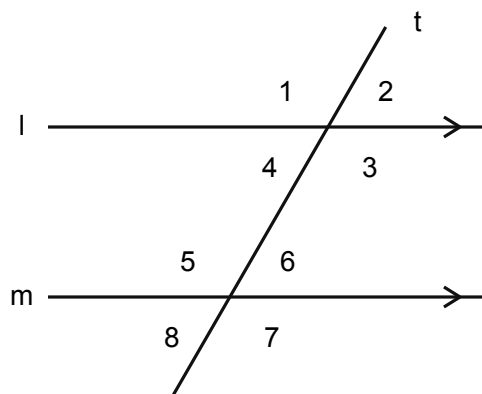
PRACTICE 4 problems

- In the worked example diagram, $m(\angle AOB) = 2x + 20$ and $m(\angle COD) = 3x - 10$ degrees. State the theorem that makes these equal, solve for x and find $m(\angle BOC)$. Justify the last step.

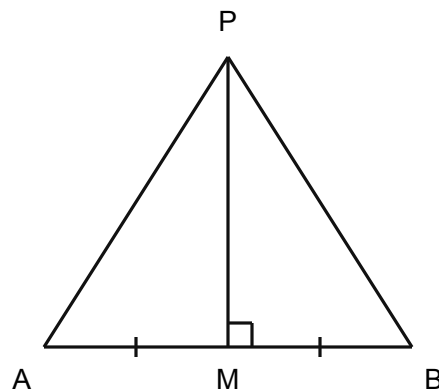
- 2** Given l parallel to m and angle $2=60$ degrees, find angles $6, 4, 8$ and 5 . For each, cite a specific angle pair and its theorem; include alternate exterior and consecutive interior relationships.



- 3** Given l parallel to m , prove alternate interior angles 3 and 5 are congruent. Use corresponding angles and vertical angles as earlier results; do not use the alternate-interior theorem as its own proof.

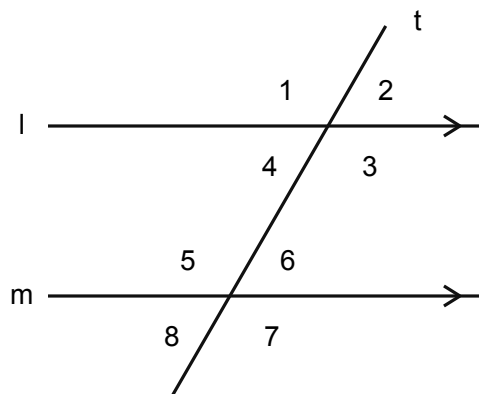


- 4** $AM=MB$ and PM is perpendicular to AB . Prove $PA=PB$ using triangles AMP and BMP . State the triangle-congruence rule and the reason the final sides agree.



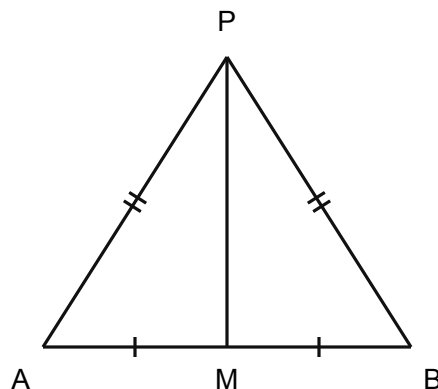
APPLY IT Show your work

- 1** Two parallel rails l, m are crossed by support t . Angles 4 and 5 measure $(2x+20)$ and $(5x+20)$ degrees. Explain why the expressions add to 180, solve for x and find angle 3.



2

A relay P is equally far from endpoints A and B. M is the midpoint of AB; draw PM as shown. Prove PM is perpendicular to AB using SSS, congruent adjacent angles and their sum. The right angle is NOT a given.



CHALLENGE One step up

P is off line AB and M is its midpoint. Explain BOTH directions of: $PA=PB$ if and only if PM is perpendicular to AB. Name the triangle-congruence rule in each direction. Then refute: " $PA=PB$ forces P to be the midpoint of AB."

PRACTICE 1

Vertical angles are congruent: $2x+20=3x-10$, so $x=30$. $\angle AOB=\angle COD=80$ degrees; $\angle BOC=100$ degrees because $\angle AOB$ and $\angle BOC$ form a linear pair.

PRACTICE 2

$6=60$ by corresponding angles 2 and 6. $4=60$ by vertical angles 2 and 4. $8=60$ by alternate exterior angles 2 and 8. $5=120$ since consecutive interior angles 4 and 5 sum to 180.

PRACTICE 3

3 is congruent to 7 by corresponding angles. 7 is congruent to 5 by vertical angles. Therefore 3 is congruent to 5 by transitivity.

PRACTICE 4

$AM=MB$ (given), $PM=PM$ (reflexive), and $\angle AMP=\angle BMP=90$ degrees (perpendicular lines). Triangles $\triangle AMP$ and $\triangle BMP$ are congruent by SAS. $PA=PB$ by corresponding parts of congruent triangles.

APPLY IT 1

4 and 5 are consecutive interior angles, so $7x+40=180$ and $x=20$. Angle 4 = 60; angle 5 = 120. Angle 3 = 120 by a linear pair with 4 (or alternate interior with 5).

The parallel rails are the given condition that licenses the consecutive-interior theorem.

APPLY IT 2

$PA=PB$ and $AM=MB$ (given); $PM=PM$ (reflexive). Triangles $\triangle AMP$ and $\triangle BMP$ are congruent by SSS, so $\angle AMP=\angle BMP$. They form a linear pair, sum to 180, and each is 90. Thus PM is perpendicular to AB . This proves the converse of the perpendicular-bisector theorem without assuming a right angle.

CHALLENGE

If PM is perpendicular, use $AM=MB$, shared PM and right included angles: SAS gives $PA=PB$. If $PA=PB$, use the three equal side pairs: SSS gives equal adjacent angles, hence two right angles. P is off AB , so it is not the midpoint; equidistance places P on the perpendicular bisector, not necessarily on AB .

Accept paragraph, flow or two-column proofs with complete reasons. Do not award a proof based only on visual appearance. In Practice 3, invoking the theorem being proved is circular. The converse requires SSS and the linear pair; a right angle cannot be assumed from the drawing.