

Input or Output?

Transform function graphs and explain what changes.

HIGH SCHOOL • HSF-BF.B.3

CCSS.Math.Content.HSF-BF.B.3 "Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs..."

NAME _____

DATE _____

Materials and setup

Pencil, straightedge and a graphing calculator or graphing app for Apply It 1. The small piecewise-linear function f in Practice 1-4 consists only of the two connected segments through $(0, 1)$, $(2, 3)$, $(4, 1)$. Draw transformed graphs with dashed segments and label them.

THE BIG IDEA

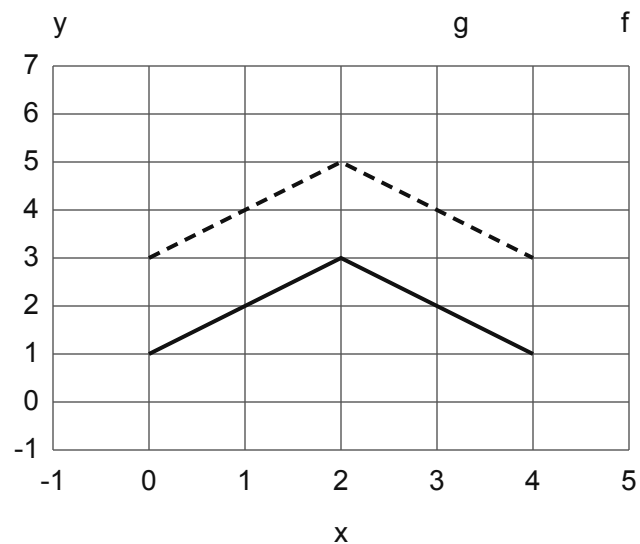
If (a, b) is on f , then $f(x) + k$ sends it to $(a, b + k)$; $kf(x)$ to (a, kb) ; $f(kx)$ to $(a/k, b)$, for $k \neq 0$; and $f(x + k)$ to $(a - k, b)$. Negative multipliers reflect across an axis. An even function has $f(-x) = f(x)$; an odd function has $f(-x) = -f(x)$, on a domain symmetric about 0.

WORKED EXAMPLE

Draw $g(x) = f(x) + 2$ for the shown two-segment function.

1. Keep the x -coordinates and add 2 to each y -coordinate.
2. The three vertices become $(0, 3)$, $(2, 5)$, $(4, 3)$.
3. Connect them in the same order; the dashed graph is 2 units higher.

Answer: This is a vertical translation up 2 units.

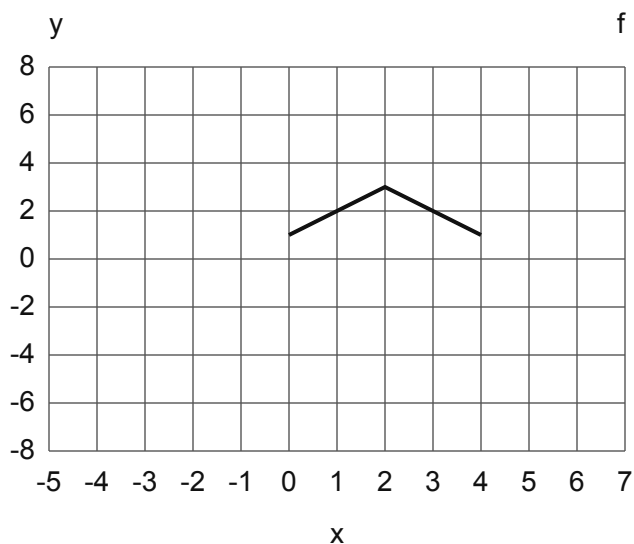


PRACTICE 4 problems

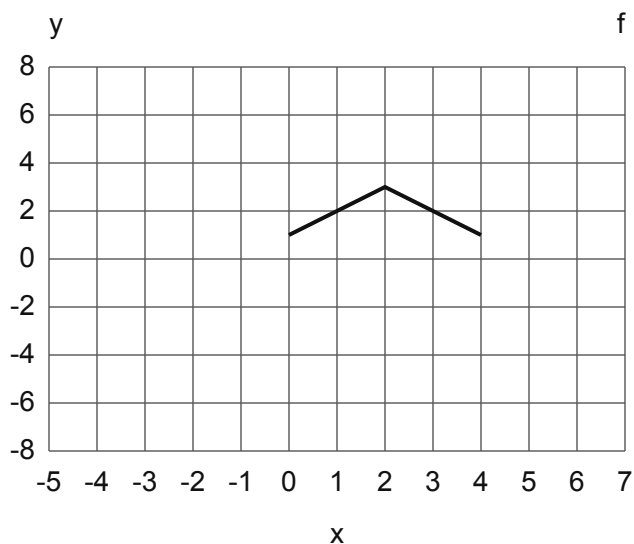
- 1** For $g(x) = f(x) - 2$, give its three vertices and describe how the graph moves. Use the given vertices of f in the materials.

ANSWER _____

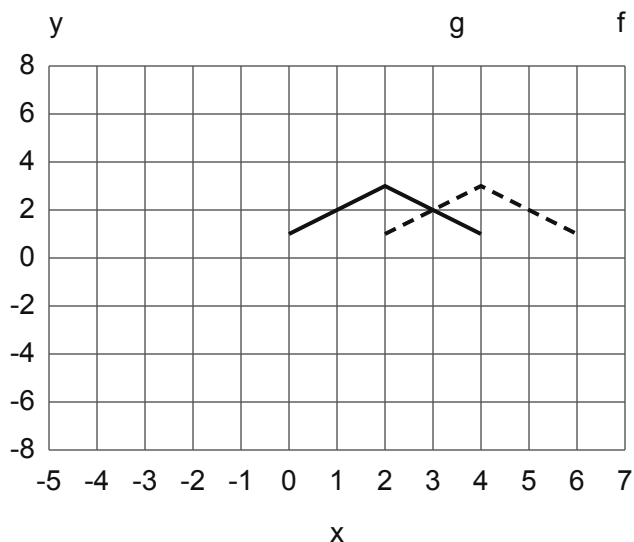
- 2** On the provided grid showing f , draw $g(x) = -2f(x)$. List its vertices on the ruled lines and explain the reflection and scale change.



- 3** On the provided grid showing f , draw $g(x) = f(-2x)$. Use the ruled lines to list its vertices and explain why the horizontal scale factor is $1/2$, not 2.



- 4** The graph labeled g is $g(x) = f(x + k)$. Read the corresponding peak positions to find k . Then state where the peak would move for $k = 3$.

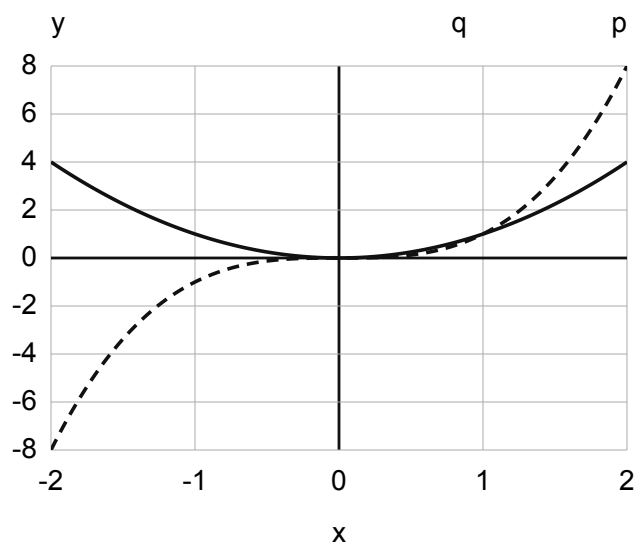


APPLY IT Show your work

- 1** Use technology with $F(x) = x^2 + x$ on the real numbers. For each form $F(x)+k$, $kF(x)$, $F(kx)$, $F(x+k)$, compare $k = 2$ and $k = -2$. Record the movement or reflection/stretch for each pair and one transformed point from $F(1) = 2$. Keep the input and output changes distinct.

2

The displayed portions of $p(x) = x^2$ (solid) and $q(x) = x^3$ (dashed) continue for all real x . Use the graph symmetries and algebra to show which function is even and which is odd.

**CHALLENGE** One step up

For $F(x) = x^2 + x$, compare $F(2x)$ with $2F(x)$. Expand both and use $x = 1$ to show they are different. Describe the two graph changes.

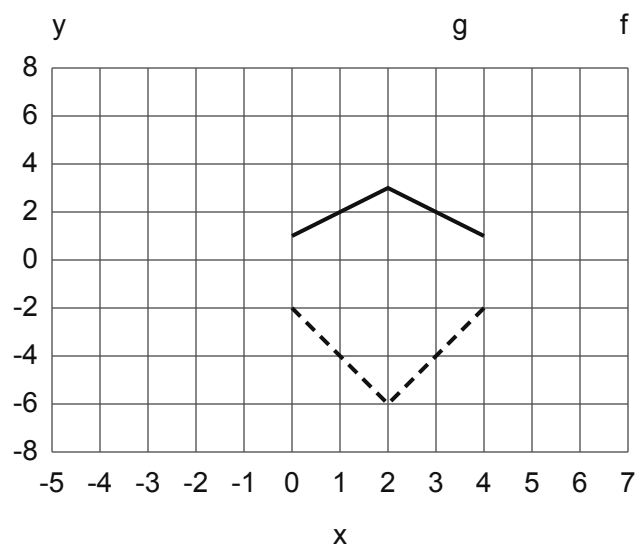
Use vertex mappings to check segment graphs; original f is defined only on $[0, 4]$, so transformed domains move too. Technology work uses the separately defined real-domain F . Require graph observations plus transformed coordinates. Parity needs both a symmetric domain and the stated algebraic identity.

PRACTICE 1

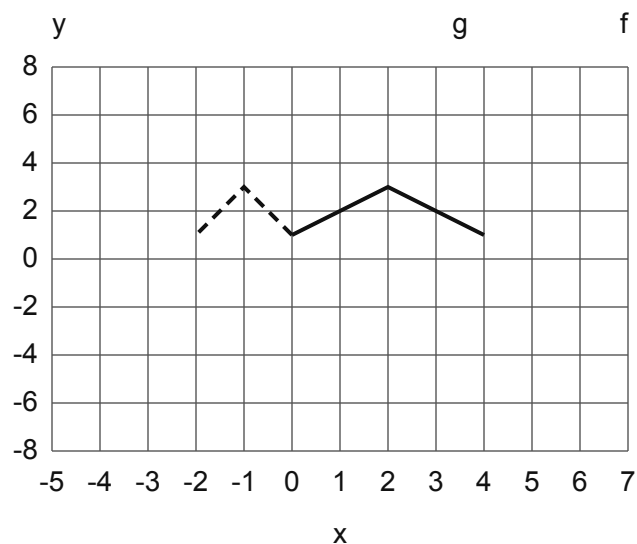
$(0, -1), (2, 1), (4, -1)$; down 2 units.

PRACTICE 2

$(0, -2), (2, -6), (4, -2)$. Reflect across the x -axis and stretch vertically by 2.

**PRACTICE 3**

$(0, 1), (-1, 3), (-2, 1)$. Each original input a becomes $a/(-2)$: reflect across the y -axis and compress horizontally by $1/2$.

**PRACTICE 4**

The peak moves from $(2, 3)$ to $(4, 3)$, right 2, so $k = -2$. For $k = 3$ it moves left 3 to $(-1, 3)$.

APPLY IT 1

$F(x)+k$: up/down 2, $(1, 4)/(1, 0)$. $kF(x)$: vertical stretch 2, with x-axis reflection for -2, $(1, 4)/(1, -4)$.
 $F(kx)$: horizontal compression 1/2, plus y-axis reflection for -2, giving $(1/2, 2)/(-1/2, 2)$. $F(x+k)$: left/right 2, giving $(-1, 2)/(3, 2)$. Students should confirm these point movements on their displayed graphs.

APPLY IT 2

p is even: y-axis symmetry; q is odd: origin symmetry.
 $p(-x)=(-x)^2=x^2=p(x)$. $q(-x)=(-x)^3=-x^3=-q(x)$. Each domain is all real numbers and is symmetric about 0.

CHALLENGE

$F(2x) = 4x^2 + 2x$, so $F(2)=6$ at $x=1$; $2F(x)=2x^2+2x$ gives 4 at $x=1$. The first compresses horizontally by 1/2; the second stretches vertically by 2. Input and output multiplication are generally different.