

Circle Angle Connections

Construct triangle circles and prove a circle-angle theorem.

HIGH SCHOOL • HSG-C.A.3

CCSS.Math.Content.HSG-C.A.3 "Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle."

NAME _____

DATE _____

Materials and setup

Pencil, compass and straightedge. For constructions, leave your compass arcs visible and label the center. Use the actual printed triangle as the given figure. Do not estimate centers by eye. Proofs must give reasons, not only angle answers.

THE BIG IDEA

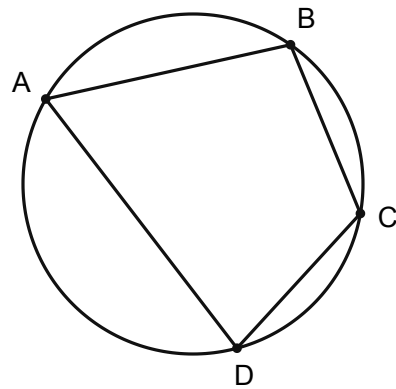
A circumcenter is the intersection of perpendicular bisectors of triangle sides; it is equidistant from the vertices. An incenter is the intersection of angle bisectors; it is equidistant from the sides. An inscribed angle measures half the arc it intercepts. Use these facts to construct the circles and prove the cyclic angle relationship.

WORKED EXAMPLE

Arc BCD measures 130° . Find the inscribed angle BAD.

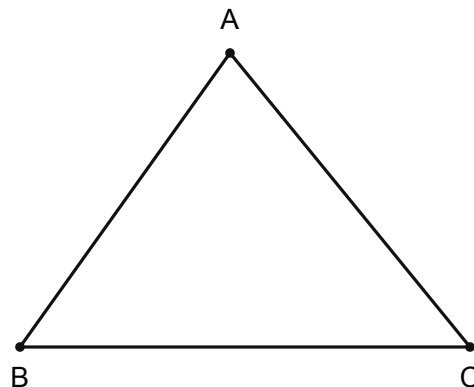
1. The vertex A is on the circle, and its rays meet the circle at B and D.
2. Angle BAD intercepts arc BCD, the arc from B to D that does not contain A.
3. An inscribed angle is half its intercepted arc: $130^\circ \div 2 = 65^\circ$.

Answer: Angle BAD measures 65° .

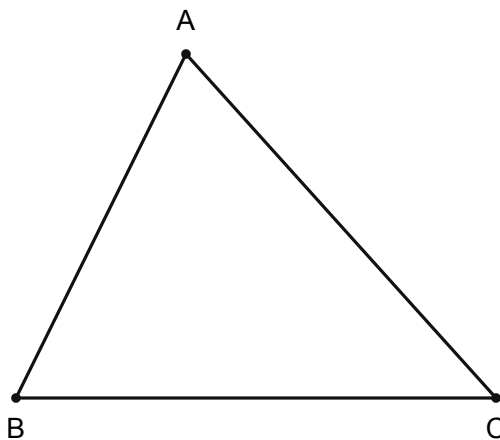


PRACTICE 4 problems

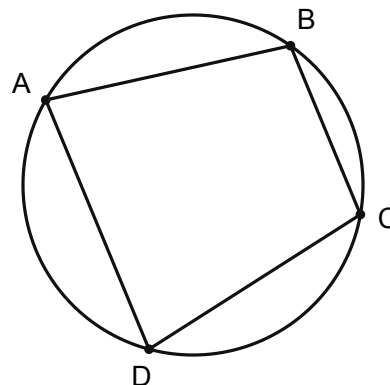
- 1** Construct the circumcircle of triangle ABC. Leave arcs, label the center O, and explain why your radius reaches all three vertices.



- 2** Construct the incircle of this triangle. Label I and the perpendicular foot T on BC. Explain why IA is not the radius of the incircle.



- 3** ABCD is inscribed in the circle. Arc BCD has measure u and the other arc BAD has measure v . Prove that angles BAD and BCD are supplementary. Use the intercepted-arc theorem and $u + v = 360^\circ$.



- 4** For a cyclic quadrilateral, $m\angle A = 3x + 12$ and $m\angle C = 5x + 8$. Find x and both angles. State the theorem that justifies your equation.

APPLY IT Show your work

- 1** A designer wants a circular border through all three corners of a triangular sign. Another wants a circle touching all three edges inside the sign. For each, state the construction lines used to locate the center and how to choose the radius.

2

Four posts W, X, Y, Z lie on a circular stage in that order. Angle W is 104° . A student claims angle Y is also 104° because both angles are inscribed. Find Y and explain why equal angles are not justified.

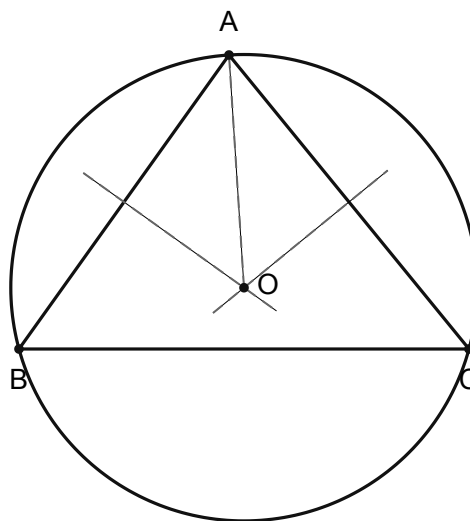
CHALLENGE One step up

The perpendicular bisectors of AB and AC meet at O. Prove that O also lies on the perpendicular bisector of BC. Then explain why a single circle centered at O passes through A, B and C.

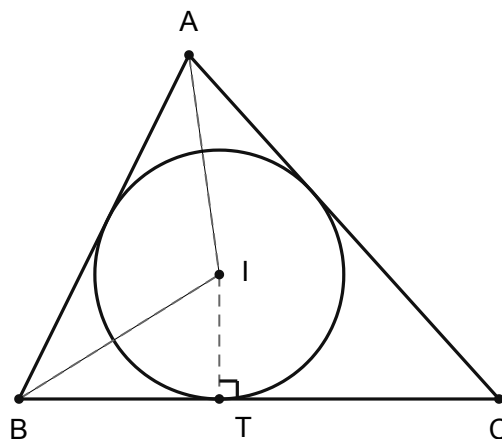
Construction keys show the finished circle and principal auxiliary lines. Require students to leave compass arcs: perpendicular bisectors use equal-radius arcs from side endpoints; angle bisectors use equal-radius arcs from points on the angle rays. Judge incidences and reasons, allowing normal compass/printing tolerance. Cyclic proofs must invoke the two intercepted arcs.

PRACTICE 1

Construct perpendicular bisectors of AB and AC; their intersection is O. Draw the circle centered at O through A. Perpendicular-bisector facts give $OA = OB = OC$.

**PRACTICE 2**

Bisect angles A and B to locate I; construct IT perpendicular to BC and draw the circle with radius IT. I has equal perpendicular distances to all sides. IA goes to a vertex, not a tangent side.

**PRACTICE 3**

$m\angle BAD = u/2$ and $m\angle BCD = v/2$. Their sum is $(u+v)/2 = 360^\circ/2 = 180^\circ$. The arcs cover the circle without overlapping interiors.

PRACTICE 4

Opposite cyclic angles are supplementary: $(3x+12)+(5x+8)=180$. Thus $x=20$, $\angle A=72^\circ$, $\angle C=108^\circ$.

APPLY IT 1

Through corners: circumcircle. Inside and touching edges: incircle.

Circumcircle: intersect perpendicular bisectors, then use the distance to any vertex. Incircle: intersect internal angle bisectors, then construct the perpendicular distance to any side. Do not use a vertex distance for the incircle.

APPLY IT 2

Angle Y is 76° .

Opposite cyclic angles sum to 180° , so $180^\circ - 104^\circ = 76^\circ$. Their intercepted arcs together make the full 360° circle; the angles are half those arc measures. Merely being inscribed does not make angles equal.

CHALLENGE

O on the first two bisectors gives $OA=OB$ and $OA=OC$. Therefore $OB=OC$. By the converse perpendicular-bisector theorem, O lies on the bisector of BC. Taking radius OA reaches all three vertices.