

Touch Point Paths

Construct tangent lines from an outside point.

HIGH SCHOOL • HSG-C.A.4

CCSS.Math.Content.HSG-C.A.4 "(+) Construct a tangent line from a point outside a given circle to the circle."

NAME _____

DATE _____

Materials and setup

Pencil, compass and straightedge. Leave compass arcs visible. To bisect OP , draw equal-radius arcs from O and P above and below OP ; join their intersections to locate M . Extend each constructed tangent through P and its touch point. Dashed curves in the worked diagram are auxiliary constructions.

THE BIG IDEA

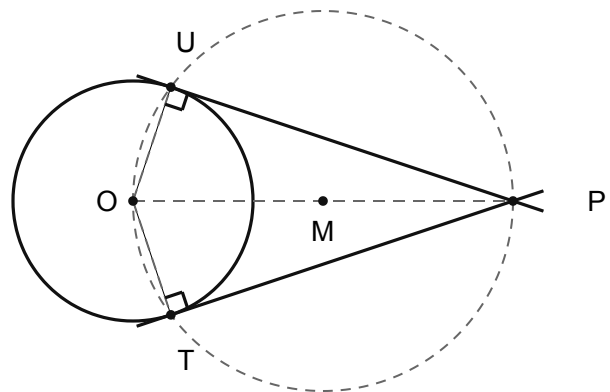
To construct tangents from an outside point P to a circle with center O , first construct the midpoint M of OP . Draw the auxiliary circle with diameter OP . Its intersections T and U with the given circle locate tangent points. Because an angle subtending a diameter is a right angle, OT is perpendicular to PT (and OU to PU).

WORKED EXAMPLE

Read the finished diagram. What roles do the solid and dashed circles play, and how are the touch points located?

1. The solid circle centered at O is the given circle; its radius fixes the boundary to be touched.
2. The dashed circle is an auxiliary construction centered at midpoint M . O and P are its diameter endpoints.
3. T and U lie on both circles. Joining each to P gives the two solid tangent lines.

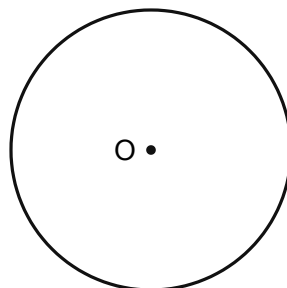
Answer: The circles locate T and U ; the right-angle marks verify the tangent directions.



PRACTICE 4 problems

- 1** Construct both tangent lines from P to the circle. Label M, T and U and keep your compass arcs. Which segment is the auxiliary circle's diameter?

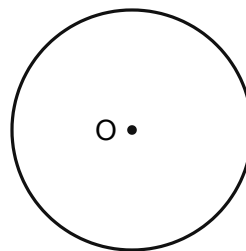
ANSWER _____



• P

- 2** Repeat the construction for this slanted OP. Label M, T and U. State a right angle that verifies a tangent.

ANSWER _____



•

P

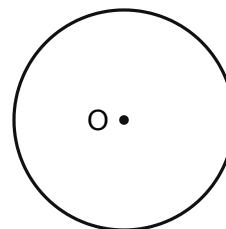
- 3** Explain why using a circle centered at P with radius PO is not the same construction. What must the auxiliary circle have as its diameter, and why?

- 4** Prove that the construction produces a tangent at T. Start with T lying on the circle with diameter OP, then relate PT to the original circle centered at O.

APPLY IT Show your work

- 1** The circle is a keep-out zone and P is a starting point. Construct the two straight paths through P that just touch the zone. Label each touch point and explain how your construction avoids crossing the interior.

P



- 2** In a correct construction, OP is 13 cm and the original radius OT is 5 cm. Find the tangent segment length PT. Show how the construction gives the right triangle needed for your calculation.

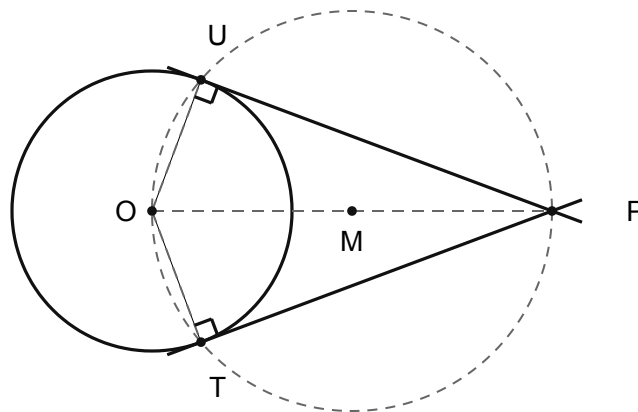
CHALLENGE One step up

How many tangent lines through P exist if P moves onto the original circle? What if P moves strictly inside it? Justify your answers using perpendicular distance from the center to a tangent line.

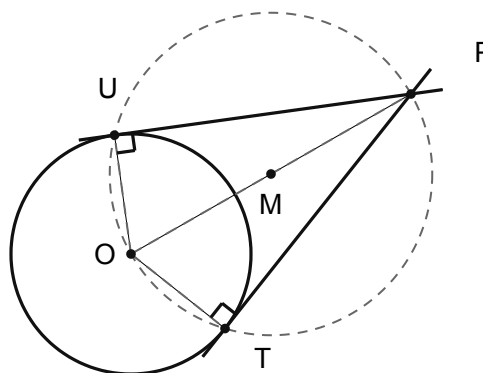
Require construction evidence, two correct touch points and tangent lines through P. Keys show the diameter circle and radius/tangent perpendicularity; students should retain their midpoint compass arcs. Allow small drawing tolerance. The extension questions do not replace the core compass-and-straightedge construction.

PRACTICE 1

Construct midpoint M of OP, then the circle centered at M through O and P. Its intersections T,U give tangent lines PT,PU. OP is the diameter.

**PRACTICE 2**

The circle with diameter OP meets the given circle at T,U. Draw PT,PU. $\angle OTP$ and $\angle OUP$ are right angles, so the lines are tangent.

**PRACTICE 3**

Its diameter must be OP. A circle centered at P makes P the center, so OP is a radius; it does not place O and P at diameter endpoints. The required right-angle conclusion at T would not follow.

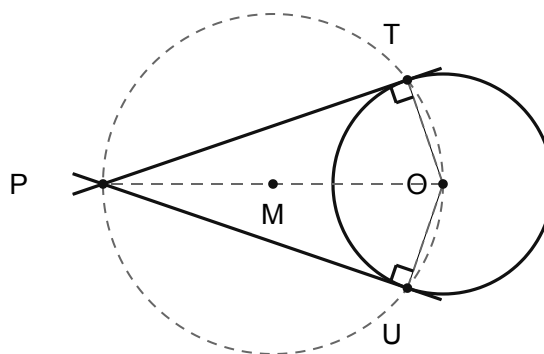
PRACTICE 4

Since T is on the circle with diameter OP, $\angle OTP = 90^\circ$. Thus PT is perpendicular to radius OT at its endpoint on the original circle. A line perpendicular to a radius at that endpoint is tangent.

APPLY IT 1

Construct the circle with diameter OP ; draw the two lines from P through its intersections with the zone boundary.

At each intersection T , the angle OTP is right. A line perpendicular to OT at T has exactly one point in common with the given circle, so it does not enter the interior.



APPLY IT 2

$PT = 12$ cm.

The diameter construction makes $\angle OTP = 90^\circ$. Thus $PT^2 = OP^2 - OT^2 = 13^2 - 5^2 = 144$, and the positive length is 12.

CHALLENGE

On the circle: one tangent, perpendicular to OP at P . Inside: none. Any line through an interior P has distance from O at most OP , which is less than the radius; a tangent must have distance exactly equal to the radius.