

Geometric Proof Toolkit

Build connected proofs about lines, triangles and parallelograms.

HIGH SCHOOL • HSG-CO.C

CCSS.Math.Content.HSG-CO.C "Prove geometric theorems"

NAME _____

DATE _____

Materials and setup

Pencil and straightedge. Write short paragraph proofs or numbered statement-and-reason chains. You may use vertical-angle, alternate-interior-angle, triangle-congruence and perpendicular-bisector theorems unless the question asks you to prove that fact. Do not use a theorem as its own proof.

THE BIG IDEA

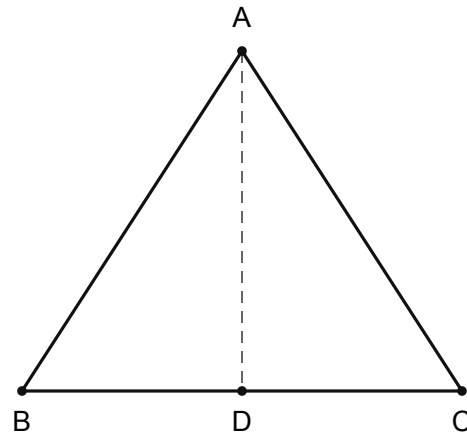
A proof connects given information to a conclusion through definitions, established theorems and logical steps. A picture suggests relationships but does not prove them. When using triangle congruence, name corresponding vertices in matching order and then justify conclusions with corresponding parts of congruent triangles.

WORKED EXAMPLE

Given $AB = AC$, prove $\angle ABC = \angle ACB$. Let AD bisect $\angle BAC$, with D on BC .

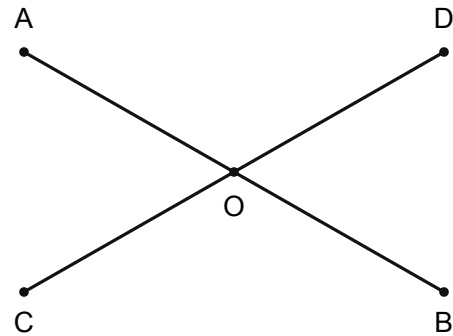
1. $AB = AC$ is given; $\angle BAD = \angle DAC$ follows from the angle bisector.
2. $AD = AD$ by the reflexive property, so triangles ABD and ACD are congruent by SAS.
3. Corresponding base angles are congruent. Since D lies on BC , they are $\angle ABC$ and $\angle ACB$.

Answer: Equal sides imply equal opposite base angles; the congruence argument proves it.

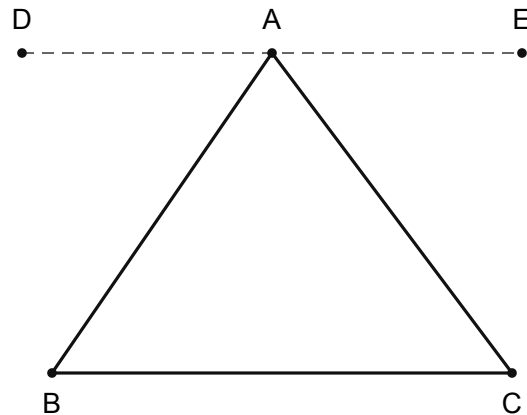


PRACTICE 4 problems

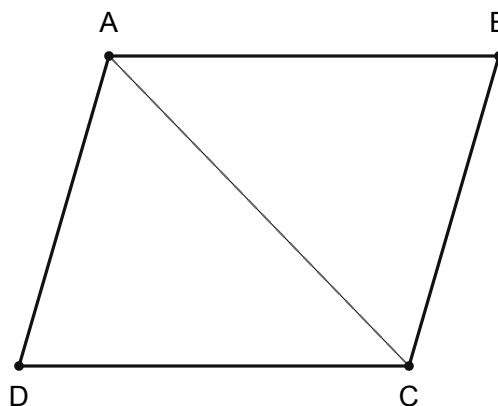
- 1** Lines AB and CD intersect at O. Prove $\angle AOC = \angle BOD$ using linear pairs. You may not cite the vertical-angle theorem as your reason.



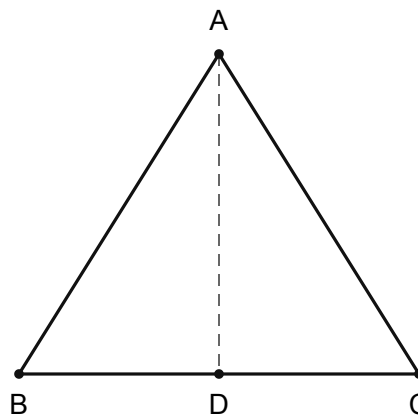
- 2** Through A draw DE parallel to BC, with D left of A and E right of A. Prove that the three interior angles of triangle ABC sum to 180° . Do not use the triangle-angle-sum theorem as a premise.



- 3** ABCD is a parallelogram; diagonal AC is drawn. Prove $AB = CD$ and $BC = AD$ using congruent triangles. State the correspondence carefully.



- 4** In triangle ABC, $AB = AC$ and D is the midpoint of BC. Prove AD is perpendicular to BC. Use congruence and a linear pair, not a measurement of the picture.



APPLY IT Show your work

- 1** Two light poles A and B are the same distance from a proposed control box P. Let M be the midpoint of AB. Assuming P is not on AB, prove PM is perpendicular to AB using congruent triangles.

- 2** A frame is a rectangle ABCD. Prove its diagonals AC and BD are congruent. Use triangles ABC and BAD; give every equality needed for congruence.

CHALLENGE One step up

A student tries to prove the isosceles base-angle theorem: " $AB = AC$, so $\angle B = \angle C$ by the isosceles theorem." Explain why this is circular. Give a valid proof strategy and identify the extra segment needed.

This cluster sampler proves representative line, triangle and parallelogram results. Accept other valid proofs with explicit givens and reasons; a numerical example or visual estimate is insufficient. Congruence correspondence matters. In Practice 3, triangles BAC and DCA match $B \leftrightarrow D$, $A \leftrightarrow C$, $C \leftrightarrow A$. Do not accept a target theorem cited as its own proof.

PRACTICE 1

$\angle AOC$ and $\angle AOD$ sum to 180° because OC and OD are opposite rays. $\angle BOD$ and $\angle AOD$ also sum to 180° because OB and OA are opposite rays. Subtract the common angle AOD to obtain equal angle measures.

PRACTICE 2

Alternate interior angles give $\angle DAB = \angle ABC$ and $\angle CAE = \angle ACB$. The three adjacent angles DAB, BAC and CAE form straight angle DAE, so their measures sum to 180° . Substitute the equal angles to get $\angle ABC + \angle BAC + \angle ACB = 180^\circ$.

PRACTICE 3

$AB \parallel CD$ gives $\angle BAC = \angle DCA$; $BC \parallel AD$ gives $\angle BCA = \angle DAC$. $AC=CA$ is common. Thus triangles BAC and DCA are congruent by ASA, so $AB=CD$ and $BC=AD$ by corresponding parts.

PRACTICE 4

$AB=AC$, $BD=CD$ and $AD=AD$ give triangles ABD and ACD congruent by SSS. Hence $\angle ADB = \angle ADC$. They form a linear pair, so each is 90° ; therefore $AD \perp BC$.

APPLY IT 1

Triangles PMA and PMB are congruent by SSS.

$PA=PB$ is given, $MA=MB$ because M is the midpoint, and PM is common. Corresponding angles PMA and PMB are equal and form a linear pair. Each is 90° , so $PM \perp AB$.

APPLY IT 2

Triangles ABC and BAD are congruent by SAS, so $AC = BD$.

$AB=BA$ is common; $BC=AD$ because a rectangle is a parallelogram. $\angle ABC$ and $\angle BAD$ are right angles, hence congruent. SAS gives the triangle congruence, and corresponding diagonal sides are equal.

CHALLENGE

The argument assumes the theorem it is supposed to prove. Construct the internal angle bisector AD with D on BC. Use $AB=AC$, $\angle BAD = \angle DAC$ and $AD=AD$ to prove triangles ABD and ACD congruent by SAS, then conclude equal base angles by corresponding parts.