

Shape Shift Paths

Explain similarity by mapping every vertex with one sequence.

GRADE 8 • 8.G.A.4

CCSS.Math.Content.8.G.A.4 "Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures..."

NAME _____

DATE _____

Materials and setup

Use a pencil, ruler and the supplied coordinate grids. Solid triangles are ABC; dashed target triangles are PQR. Match A to P, B to Q and C to R. For a drawing task, label the new vertices and connect them.

FOLLOW THE WHOLE FIGURE

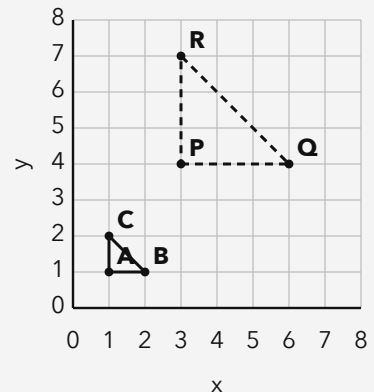
Two figures are similar when a sequence of rotations, reflections, translations and dilations maps one onto the other. Rigid motions preserve lengths and angles. A dilation preserves angles and multiplies all lengths by one positive scale factor. State the order, center or mirror line, and movement for every step.

WORKED EXAMPLE

Map solid triangle ABC to dashed triangle PQR. Describe a sequence and check all three vertices.

1. Dilate by factor 3 about (0,0): (1,1), (2,1), (1,2) become (3,3), (6,3), (3,6).
2. Translate up 1 unit: the vertices become (3,4), (6,4), (3,7).
3. Every new vertex matches its target; all lengths tripled and angles stayed equal.

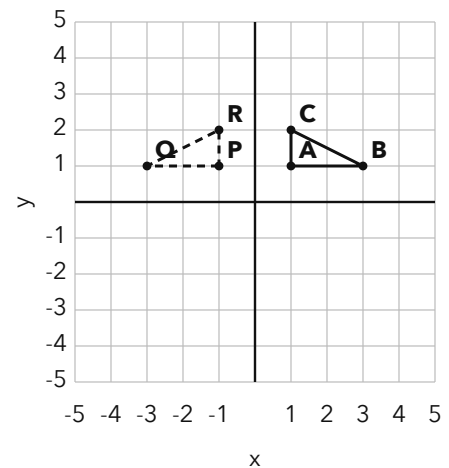
Answer: Dilate by 3 about the origin, then translate up 1 unit.



PRACTICE 4 problems

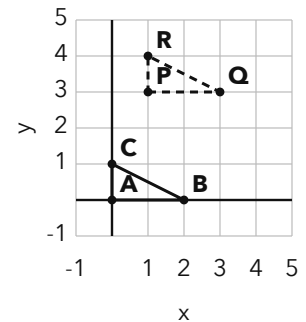
- 1** Describe the single transformation mapping ABC to PQR.
State its mirror line and check the three vertices.

ANSWER _____



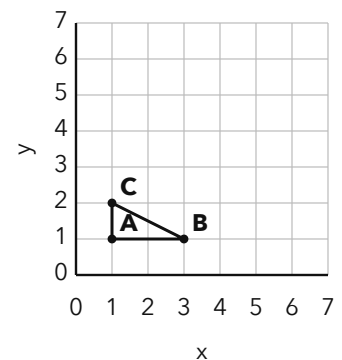
- 2** Describe the single transformation mapping ABC to PQR. Explain why the triangles are also congruent.

ANSWER _____



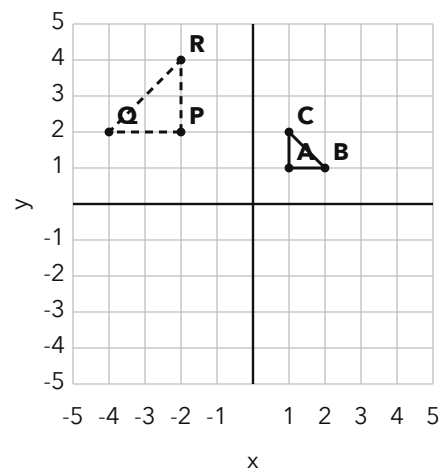
- 3** On this grid, draw the image of ABC after a dilation by factor 2 about (0,0). Label the image PQR. State whether it is congruent to ABC.

ANSWER _____



- 4** Find a two-step sequence mapping ABC to PQR. Use a reflection and a dilation about the origin; check all three target vertices.

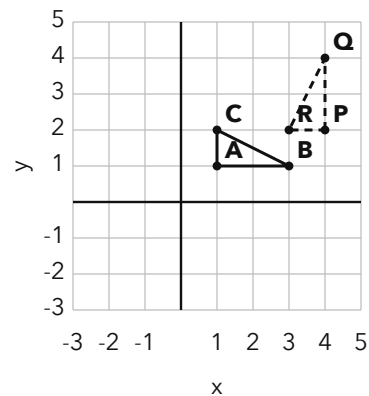
ANSWER _____



APPLY IT Show your work

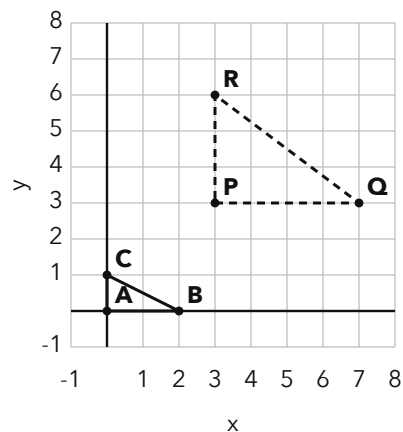
- 1** A logo designer maps solid ABC to dashed PQR. First rotate 90° counterclockwise about the origin. What translation completes the mapping? Show the intermediate coordinates.

ANSWER _____



- 2** Two triangular sign designs have the shown vertices. A designer claims they are similar. Compare the horizontal and vertical leg ratios. Can any allowed similarity sequence map ABC to PQR?

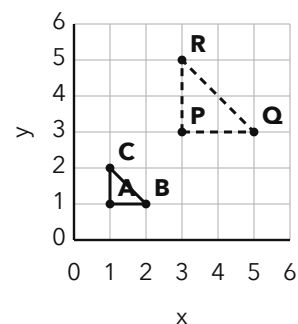
ANSWER _____



CHALLENGE One step up

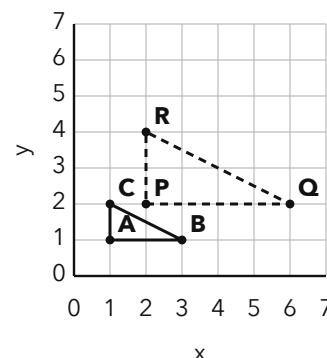
Kai wants to map ABC to PQR. He dilates by 2 about the origin, then translates right 1 and up 1. Would translating right 1 and up 1 first, then dilating by 2, reach the same target? Check A and give the correct translation to use before the dilation.

WORK SPACE AND ANSWER



PRACTICE 3

P(2,2), Q(6,2), R(2,4). Similar but not congruent: lengths double.

**PRACTICE 1**

Reflect across the y-axis: $(1,1) \rightarrow (-1,1)$, $(3,1) \rightarrow (-3,1)$, $(1,2) \rightarrow (-1,2)$.

PRACTICE 2

Translate right 1 and up 3. All vertices move by $(1,3)$; a translation preserves lengths and angles.

PRACTICE 4

Reflect across the y-axis, then dilate by 2 about the origin: $P(-2,2)$, $Q(-4,2)$, $R(-2,4)$. The reverse order also works here.

APPLY IT 1

After rotation: $(-1,1)$, $(-1,3)$, $(-2,1)$. Translate right 5 and up 1 to reach $P(4,2)$, $Q(4,4)$, $R(3,2)$.

Use $(x,y) \rightarrow (-y,x)$; the same translation $(5,1)$ must work for every vertex.

APPLY IT 2

No. $AB=2$ and $PQ=4$ give ratio 2, while $AC=1$ and $PR=3$ give ratio 3. No common scale factor fits.

The marked vertex correspondence already fails. More generally the right triangles have leg ratios 2:1 and 4:3 (or 3:4), so changing correspondence cannot make them similar.

CHALLENGE

No: A(1,1) reaches (4,4), not P(3,3), if translated by (1,1) first. Translate by (0.5,0.5) before dilation instead; doubling gives the required (1,1) shift.

Accept any valid sequence that maps all corresponding vertices. Rotation directions are viewed on the page; origin means (0,0). Require the actual labeled drawing in Practice 3. Explain non-similarity through inconsistent scale factors, not visual impressions alone.