

Scale and Slide

Connect dilations, angle preservation and triangle similarity.

HIGH SCHOOL • HSG-SRT.A

CCSS.Math.Content.HSG-SRT.A "Understand similarity in terms of similarity transformations"

NAME _____

DATE _____

Materials and setup

Use a pencil and ruler. Solid triangles are ABC and dashed target triangles are PQR, with $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$ unless stated otherwise. Give reasons, not only numerical answers.

WHAT A DILATION PRESERVES

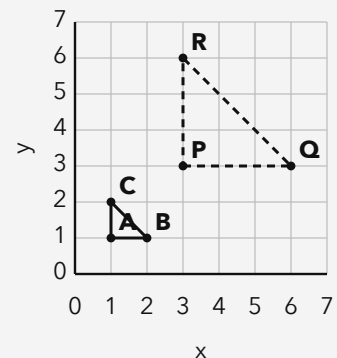
A dilation with center O and positive scale factor k sends X along ray OX so $OX' = k \cdot OX$. It multiplies every length by k and preserves angles. A line through O maps onto itself. For $k \neq 1$, a line not through O maps to a distinct parallel line; $k = 1$ leaves everything unchanged. Similarity combines a dilation with rigid motions. Similar triangles have equal corresponding angles and proportional corresponding sides.

WORKED EXAMPLE

The dashed triangle is the dilation of ABC by factor 3 about the origin. Compare line AB with line PQ, then compare line AC with line PR.

1. Line AB is $y=1$; its image line PQ is $y=3$. Both are horizontal.
2. Line AC is $x=1$; its image line PR is $x=3$. Both are vertical.
3. Neither original line passes through the dilation center; each image is a distinct parallel line.

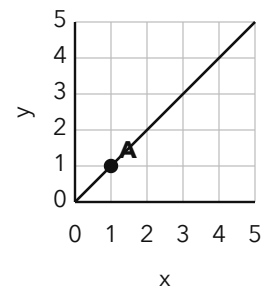
Answer: $AB \parallel PQ$ and $AC \parallel PR$. The corresponding segment lengths are tripled.



PRACTICE 4 problems

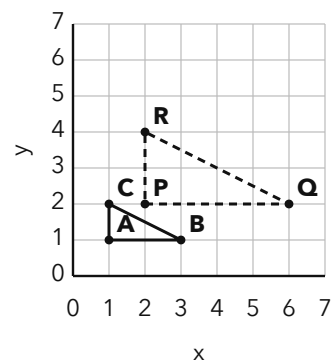
- 1 A dilation of factor 3 about the origin maps the line through (0,0) and (1,1). Locate the images of these points. Does the whole line change? Explain.

ANSWER _____



- 2** Read both triangles. Compare AB with PQ and AC with PR. Determine the scale factor and explain why angle A equals angle P.

ANSWER _____



- 3** One triangle has side lengths 3, 4 and 5. A second has corresponding sides 6, 8 and 11. Are they similar? Justify with a condition that every corresponding side must satisfy.

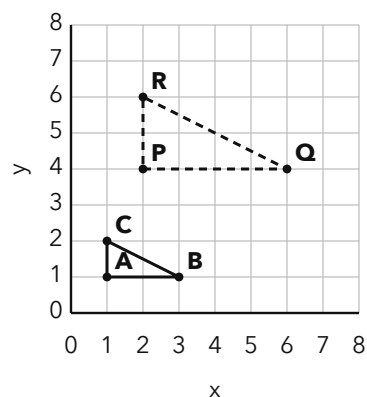
ANSWER _____

- 4** Triangles ABC and DEF have $\angle A = \angle D = 40^\circ$ and $\angle B = \angle E = 65^\circ$. Find the third angles and explain the AA similarity conclusion. Does AA determine one size?

ANSWER _____

APPLY IT Show your work

- 1** A designer maps ABC to PQR. Find a dilation about the origin followed by a translation that works. Verify every vertex and name two properties this sequence preserves.



ANSWER _____

2

Explain why AA works using transformations. Suppose $\angle A = \angle D$ and $\angle B = \angle E$ in nondegenerate triangles ABC and DEF. Describe a dilation and rigid motions that make A match D and B match E. Then explain why C must match F.

ANSWER _____

CHALLENGE One step up

A dilation has positive scale factor k . Use a pair of points (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ to show why a nonvertical line and its image have equal slopes. Explain separately what happens to a vertical line. Take the dilation center to be the origin.

PRACTICE 1

Images (0,0) and (3,3) lie on $y=x$. The entire line maps onto itself because it passes through the center.

PRACTICE 2

$AB=2$, $PQ=4$; $AC=1$, $PR=2$; both ratios are 2. A dilation about the origin produces PQR and preserves every angle, so $\angle A = \angle P$.

PRACTICE 3

No. The ratios $6/3$ and $8/4$ are 2, but $11/5$ is 2.2. No common scale factor fits all three.

PRACTICE 4

Both third angles are 75° . Two matching angles establish the same shape by AA, so $ABC \sim DEF$. The scale factor is not determined; many sizes are possible.

APPLY IT 1

Dilate by 2, then translate up 2: $(1,1) \rightarrow (2,4)$, $(3,1) \rightarrow (6,4)$, $(1,2) \rightarrow (2,6)$. Angle measures and ratios of corresponding lengths are preserved.

All lengths scale by 2; translation adds no further scale. The figures are similar, not congruent.

APPLY IT 2

Dilate ABC by DE/AB . Rigidly move A to D and B to E; reflect across DE if needed to put the third vertices on the same side. The equal angles at D and E place the third vertex at the intersection of the same two rays, namely F.

Two distinct nonparallel rays meeting on the specified side have a unique intersection. This supplies the geometric reason, beyond just naming AA.

CHALLENGE

Image slope = $(k y_2 - k y_1) / (k x_2 - k x_1) = (y_2 - y_1) / (x_2 - x_1)$. A vertical line $x=a$ maps to $x=ka$, also vertical. Thus image lines have the same direction; a line through the center coincides with its image.

The cluster requires a connection between similarity transformations and geometric properties. Require the transformation argument in the second applied problem (the AA explanation), including same-side placement. Slope is undefined for vertical lines, so handle them by $x=a \rightarrow x=ka$. Exact agreement in one side ratio alone does not prove similarity.