

Triangle Proof Workshop

Prove proportionality and the Pythagorean Theorem using similarity.

HIGH SCHOOL • HSG-SRT.B.4

CCSS.Math.Content.HSG-SRT.B.4 "Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity."

NAME _____

DATE _____

Materials and setup

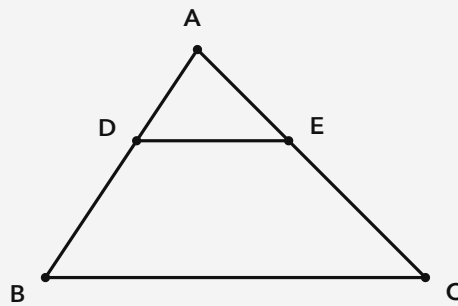
Use a pencil and ruler. Diagrams show the vertex order. In the split-side diagrams, D lies strictly between A and B, and E strictly between A and C. All triangles are nondegenerate, so side lengths and the denominators used below are positive.

FROM SIMILAR TRIANGLES TO THEOREMS

A proof uses stated assumptions and justified steps to cover every allowed triangle. A diagram guides the argument but does not establish parallelism or proportionality by appearance. Use angle relationships, AA or SAS similarity, and algebra. Do not use the theorem you are trying to prove as a reason.

WORKED EXAMPLE

Given $DE \parallel BC$, prove $AD/DB = AE/EC$. State why the smaller triangle is similar to the whole triangle.

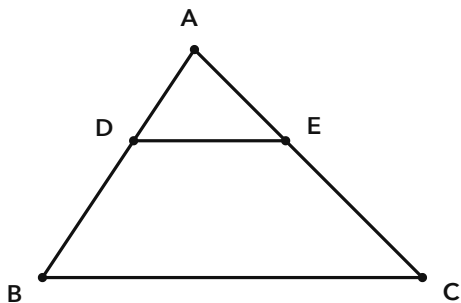


1. Corresponding angles from the parallel lines and the shared angle A give triangle ADE ~ triangle ABC by AA.
2. Thus $AD/AB = AE/AC$. Replace AB with $AD+DB$ and AC with $AE+EC$.
3. Cross-multiply: $AD(AE+EC) = AE(AD+DB)$. Subtract $AD \cdot AE$ from both sides: $AD \cdot EC = AE \cdot DB$.
4. Divide by $DB \cdot EC$, which is positive.

Answer: $AD/DB = AE/EC$. This proves the parallel-line proportionality theorem for the stated configuration.

PRACTICE 4 problems

- 1** Given $DE \parallel BC$, identify two equal angle pairs that establish $\triangle ADE \sim \triangle ABC$. Explain each equality.

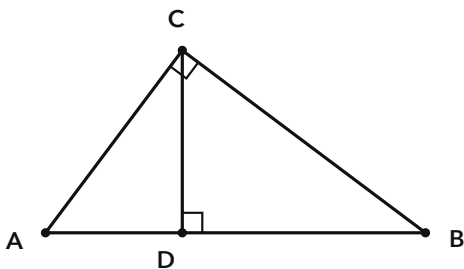


ANSWER _____

- 2** A proof starts with $AD/DB = AE/EC$. Show algebraically that $AD/AB = AE/AC$, using $AB = AD + DB$ and $AC = AE + EC$.

ANSWER _____

- 3** In right triangle ABC , CD is perpendicular to hypotenuse AB . Explain why $\triangle ABC \sim \triangle ACD$ by AA. Keep corresponding vertices in the correct order.



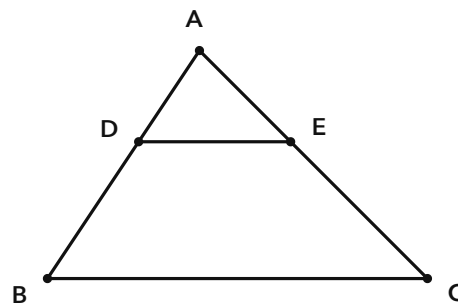
ANSWER _____

- 4** For the right-triangle configuration in question 3, prove $\triangle ABC \sim \triangle CBD$ by AA. Identify both angle pairs.

ANSWER _____

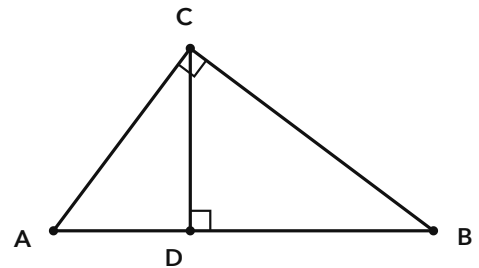
APPLY IT Show your work**1**

Prove the converse: if $AD/DB = AE/EC$, then $DE \parallel BC$. Use the ratio conversion from Practice 2, the shared angle at A, and SAS similarity. Finish by identifying the angles that force parallel lines.

**REASONING AND ANSWER**

2

Prove the Pythagorean Theorem for right triangle ABC using the two similarities from Practice 3-4. Let $AB=c$, $BC=a$, $CA=b$, $AD=x$ and $DB=y$. Derive one equation for b^2 and one for a^2 from corresponding sides. Then combine them to prove $a^2+b^2=c^2$. Do not assume the Pythagorean Theorem.



REASONING AND ANSWER

CHALLENGE One step up

A student checks three right triangles numerically and declares the Pythagorean Theorem proved. Explain why that is insufficient and identify what makes your similarity proof apply to every nondegenerate right triangle.

PRACTICE 1

$\angle DAE = \angle BAC$ is the shared angle; $\angle ADE = \angle ABC$ follows from corresponding angles of parallel lines. Alternatively use $\angle AED = \angle ACB$.

PRACTICE 2

Cross-multiply to $AD \cdot EC = AE \cdot DB$. Add $AD \cdot AE$ to get $AD(AE + EC) = AE(AD + DB)$. Divide by $AB \cdot AC$ to obtain $AD/AB = AE/AC$.

PRACTICE 3

$\angle ACB = \angle ADC = 90^\circ$ and $\angle BAC = \angle CAD$ because D lies on AB . Thus $ABC \sim ACD$; correspondence $A \leftrightarrow A$, $B \leftrightarrow C$, $C \leftrightarrow D$.

PRACTICE 4

$\angle ACB = \angle CDB = 90^\circ$ and $\angle ABC = \angle CBD$ because D lies on BA . Thus $ABC \sim CBD$, with $A \leftrightarrow C$, $B \leftrightarrow B$, $C \leftrightarrow D$.

APPLY IT 1

From Practice 2, $AD/AB = AE/AC$. The included angle DAE equals BAC . SAS gives $ADE \sim ABC$. Therefore $\angle ADE = \angle ABC$. Since AB is a transversal and these corresponding angles are equal, $DE \parallel BC$. The diagram alone is not evidence of parallelism. Parallelism is the conclusion, so do not use it to justify the initial similarity.

APPLY IT 2

$ABC \sim ACD$ gives $AC/AB = AD/AC$, so $b/c = x/b$ and $b^2 = cx$. $ABC \sim CBD$ gives $BC/AB = BD/BC$, so $a/c = y/a$ and $a^2 = cy$. Adding yields $a^2 + b^2 = c(x+y) = c \cdot AB = c^2$.

The similarities follow from AA, independently of the theorem being proved. D lies inside AB , so $x+y=c$.

CHALLENGE

Three examples cannot cover all right triangles. In any nondegenerate right triangle, the altitude to the hypotenuse creates the AA similarities used above. The argument uses arbitrary positive side lengths, so it proves the identity generally.

Assess complete chains of justified statements. Require the parallel-line theorem, its converse, and the Pythagorean proof from triangle similarity. Practice 2 supplies algebra for the converse; Practice 3–4 establish independent AA similarities. Accept other valid general proofs, but not measured examples, appearance-based parallelism or circular use of the theorem.