

Similarity Transformations

Use transformations, side ratios and angle preservation to justify similarity.

HIGH SCHOOL • HSG-SRT.A.2

CCSS.Math.Content.HSG-SRT.A.2 "Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding..."

NAME _____

DATE _____

HOW SIMILARITY WORKS

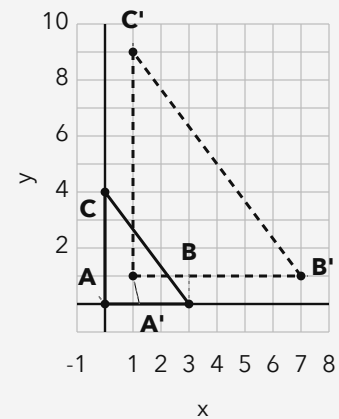
Similar figures can be matched by rigid motions and a dilation with one positive scale factor. Rigid motions preserve lengths and angles; dilation preserves angles and multiplies every length by the same factor. Match corresponding vertices in order, and describe the center and order of transformations.

WORKED EXAMPLE

The solid triangle is ABC and the dashed triangle is $A'B'C'$. $A(0,0)$, $B(3,0)$, $C(0,4)$; $A'(1,1)$, $B'(7,1)$, $C'(1,9)$. Explain a similarity transformation.

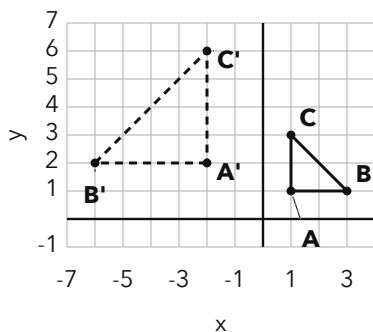
1. Dilate ABC about the origin by factor 2: the vertices become $(0,0)$, $(6,0)$, $(0,8)$.
2. Translate 1 unit right and 1 unit up. These images are exactly A' , B' and C' .
3. The sides 3, 4, 5 become 6, 8, 10. Every ratio is 2; corresponding angles stay equal.

Answer: Yes. Dilation by 2 about the origin, then translation by $(1,1)$, maps the figures.



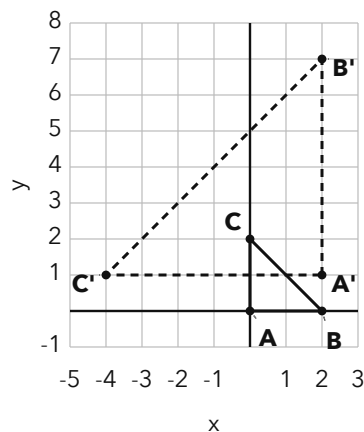
PRACTICE 4 problems

- 1** Describe a sequence mapping ABC to $A'B'C'$. Give its scale factor. $A(1,1)$, $B(3,1)$, $C(1,3)$; $A'(-2,2)$, $B'(-6,2)$, $C'(-2,6)$. Solid is original; dashed is image.



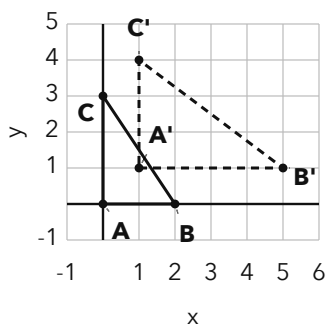
ANSWER _____

- 2** Decide similarity. Describe the motions and explain what happens to the corresponding angles. $A(0,0)$, $B(2,0)$, $C(0,2)$; $A'(2,1)$, $B'(2,7)$, $C'(-4,1)$. Solid is original; dashed is image.



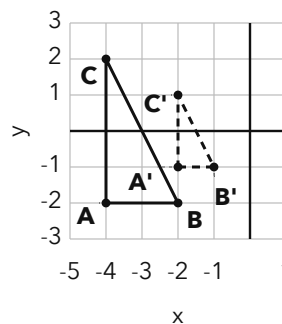
ANSWER _____

- 3** Could one similarity transformation map these triangles in the stated vertex order? Compare the horizontal and vertical side ratios. $A(0,0)$, $B(2,0)$, $C(0,3)$; $A'(1,1)$, $B'(5,1)$, $C'(1,4)$. Solid is original; dashed is image.



ANSWER _____

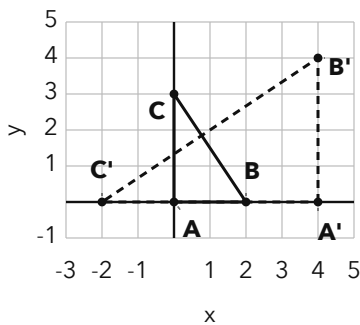
- 4** Describe the transformation and explain why a smaller image can still be similar. $A(-4,-2)$, $B(-2,-2)$, $C(-4,2)$; $A'(-2,-1)$, $B'(-1,-1)$, $C'(-2,1)$. Solid is original; dashed is image.



ANSWER _____

APPLY IT Show your work

- 1** A map icon uses ABC . An enlarged, turned copy has the listed dashed vertices. $A(0,0)$, $B(2,0)$, $C(0,3)$; $A'(4,0)$, $B'(4,4)$, $C'(-2,0)$. Find a sequence mapping every vertex, then explain why both side ratios and angles satisfy similarity.



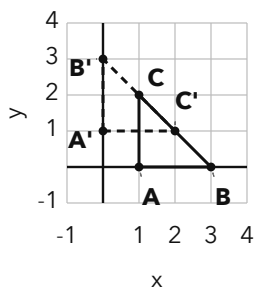
ANSWER _____

- 2** A designer scales a right-triangle logo with legs 6 and 8 and hypotenuse 10 to legs 9 and 12 and hypotenuse 15. Explain how dilation and, if needed, rigid motions can match the logos. Explain the corresponding side ratios and angles.

ANSWER _____

CHALLENGE One step up

A student says these triangles cannot be similar because the matching horizontal side becomes vertical. $A(1,0)$, $B(3,0)$, $C(1,2)$; $A'(0,1)$, $B'(0,3)$, $C'(2,1)$. Use a transformation to accept or reject the claim. Explain the side ratios and corresponding angles.



PRACTICE 1

Describe a sequence mapping ABC to $A'B'C'$.
Give its scale factor. $A(1,1)$, $B(3,1)$, $C(1,3)$; $A'(-2,2)$, $B'(-6,2)$, $C'(-2,6)$. Solid is original; dashed is image.

Reflect across the y-axis, then dilate about the origin by factor 2. Each corresponding side doubles and angles are preserved.

PRACTICE 2

Decide similarity. Describe the motions and explain what happens to the corresponding angles. $A(0,0)$, $B(2,0)$, $C(0,2)$; $A'(2,1)$, $B'(2,7)$, $C'(-4,1)$. Solid is original; dashed is image.

Rotate 90° counterclockwise about the origin, dilate there by 3, then translate $(2,1)$. Corresponding angles remain $90^\circ, 45^\circ, 45^\circ$; sides $2, 2, 2\sqrt{2}$ become $6, 6, 6\sqrt{2}$.

PRACTICE 3

Could one similarity transformation map these triangles in the stated vertex order? Compare the horizontal and vertical side ratios. $A(0,0)$, $B(2,0)$, $C(0,3)$; $A'(1,1)$, $B'(5,1)$, $C'(1,4)$. Solid is original; dashed is image.

No. $A'B'/AB=4/2=2$ but $A'C'/AC=3/3=1$. A single dilation factor cannot produce both ratios, and rigid motions cannot change them.

PRACTICE 4

Describe the transformation and explain why a smaller image can still be similar. $A(-4,-2)$, $B(-2,-2)$, $C(-4,2)$; $A'(-2,-1)$, $B'(-1,-1)$, $C'(-2,1)$. Solid is original; dashed is image.

Dilate about the origin by factor $1/2$. All sides halve and corresponding angles remain equal, so the reduction is similar.

APPLY IT 1

Rotate 90° counterclockwise about the origin, dilate there by 2, then translate 4 units right.

Rotation gives $(0,0)$, $(0,2)$, $(-3,0)$; doubling gives $(0,0)$, $(0,4)$, $(-6,0)$; translating gives $(4,0)$, $(4,4)$, $(-2,0)$. Every side doubles and corresponding angles are preserved.

APPLY IT 2

Dilate the original by $3/2$, then use any needed rotation, reflection or translation to place it on the target. $9/6=12/8=15/10=3/2$.

Dilation gives the target side lengths; a congruence motion can then align the triangles. Both operations preserve corresponding angles.

CHALLENGE

Reject the claim. Reflect across $y=x$, sending each (x,y) to (y,x) . This maps all three vertices. The scale factor is 1; corresponding side ratios are 1 and angles stay equal. Congruent figures are also similar.

Accept any correct sequence that maps all three corresponding vertices. Check the center and order of dilations; a scale factor alone is not a full coordinate mapping. Solid and dashed graphs are supplied figures to compare, not blank graphing tasks. In Practice 2 the angle measures and exact side lengths come from the same right-isosceles coordinates. Do not infer similarity from appearance alone. Rigid motions alone are allowed when the scale factor is 1.