

Build the Volume Rule

Count cubes, compare layers, develop a formula

GRADE 5 • 5.MG.2.d

5.MG.2.d "Investigate and develop a formula for determining the volume of rectangular prisms using concrete objects."

NAME _____

DATE _____

BUILD UNDERSTANDING

Use 24 identical unit cubes with no gaps or overlaps. A cube has volume 1 cubic unit. Build each model physically, then record counts. A top-view square shows one cube in a layer; it does not show the full stack. Reuse your cubes between tasks. The 0 in the challenge means an empty space.

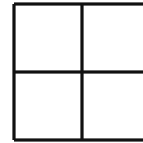
WORKED EXAMPLE

Together, build one layer 2 cubes long and 2 cubes wide. Count the cubes by rows. What is this one-layer model's volume?

1. Make two rows with two unit cubes in each.
2. Count $2 + 2 = 4$ cubes.

Answer: The one-layer model has volume 4 cubic units.

Top view: one layer

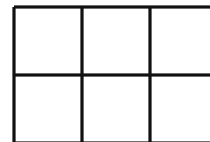


Each square is one cube top

PRACTICE 4 problems

- 1** Build a filled prism 3 cubes long, 2 wide and 2 high. Record cubes in one layer, number of layers and total cubes. Check the total by counting the actual cubes.

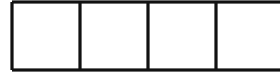
Top view: one layer



Each square is one cube top

- 2** Rebuild the same 12 cubes into a filled prism 4 cubes long, 1 wide and 3 high. Record cubes per layer, layers and total. Write a multiplication equation matching your model.

Top view: one layer

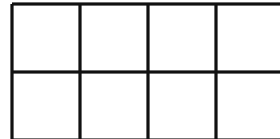


Each square is one cube top

- 3** Compare your two built prisms from Practice 1 and 2. Their shapes differ. Explain why their volumes are equal, using the cubes and layers.

- 4** Before building, predict the cubes needed for a filled prism 4 long, 2 wide and 3 high. Then build it with the 24 cubes. Record your layer calculation and whether the physical count agrees.

Top view: one layer

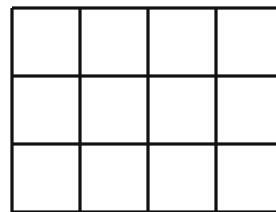


Each square is one cube top

APPLY IT Show your work**1**

A model storage case fits unit cubes 4 along its length, 3 across its width and 2 layers high. Build the filled model. Find its volume and explain how one layer helps you count all cubes.

Top view: one layer



Each square is one cube top

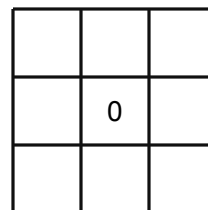
2

Use your built models to develop a rule. Let l be cubes along the length, w be rows across the width, and h be layers high. Write a formula for volume V . Explain what each multiplication counts.

CHALLENGE One step up

Build two 3-by-3 layers, leaving the center cube out of each, as shown. Stack them. Count the cubes. Why would 3 times 3 times 2 overcount this model? State when your formula works.

Top view: one layer



0 = empty; other squares = cubes

Require actual construction with identical cubes before accepting the volume rule. Observe bottom-layer counts, equal layer counts and total cubes. Volumes are in cubic units, not square units. Accept $V = (l \text{ times } w) \text{ times } h$ or an equivalent rule with an explanation. The hollow challenge model is not a filled prism.

PRACTICE 1

6 cubes per layer; 2 layers; 12 cubes, so volume is 12 cubic units. Repeated addition: $6 + 6 = 12$.

PRACTICE 2

4 cubes per layer; 3 layers; 12 cubes. 4 times 3 = 12 cubic units.

PRACTICE 3

Both contain 12 unit cubes with no gaps. The first has 6 cubes in each of 2 layers; the second has 4 in each of 3 layers. Different dimensions can give equal volume.

PRACTICE 4

One layer has 4 times 2 = 8 cubes. Three layers need 8 times 3 = 24 cubes. The built filled prism agrees.

APPLY IT 1

4 times 3 = 12 cubes per layer. Two equal layers contain 24 cubes, so volume is 24 cubic units. Accept equivalent correct reasoning with the stated units and evidence.

APPLY IT 2

$V = l \text{ times } w \text{ times } h$. $l \text{ times } w$ counts unit cubes in one layer; multiplying by h counts all equal layers. This describes a filled rectangular prism. Accept equivalent correct reasoning with the stated units and evidence.

CHALLENGE

Each layer has $9 - 1 = 8$ cubes, so 2 layers contain 16. The product 3 times 3 times 2 = 18 counts two absent cubes. The formula requires a filled rectangular prism with no gaps.