LESSON MAKER PRO

# Volume and Triangle Lab

*Explain formulas with layers, pouring and area models.*

**GRADE 8 · TEKS 8.6**

**8.6** “Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas...”

NAME ______________________ DATE ____________

## CONNECT THE REPRESENTATIONS

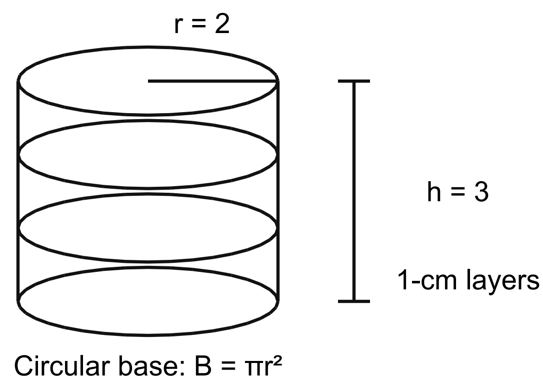
A geometric model explains why a formula works. Cylinders can be built from equal circular layers; matching cone pours reveal a volume ratio. Rearranging the same right-triangle pieces inside equal outer squares compares the areas built on their sides.

### WORKED EXAMPLE

The cylinder has r = 2 cm and h = 3 cm. Explain V = Bh by using the three congruent 1-cm layers shown. Then find its volume.

1. Each circular base has area $B = \pi(2^2) = 4\pi$ $cm^2$.
2. Each 1-cm layer occupies $4\pi$ $cm^3$. Three layers fill the 3-cm height, so V = Bh.

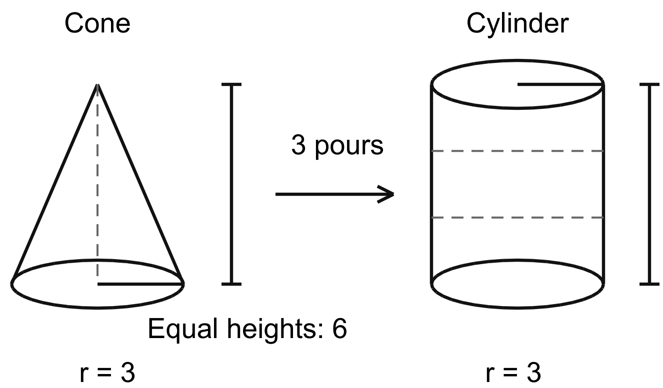
**Answer: $B = 4\pi$ $cm^2$; $V = 3(4\pi \times 1) = 12\pi$ $cm^3$.**




## PRACTICE 4 problems

**1** A cylinder has radius 3 cm and height 5 cm. Name B and explain its volume as five 1-cm layers. What happens to volume if height doubles while the base stays fixed?

**2** The congruent bases have r = 3 cm and both heights are 6 cm. In a pouring model, three full cones exactly fill the cylinder. Use the diagram and observation to explain both formulas and find both volumes.

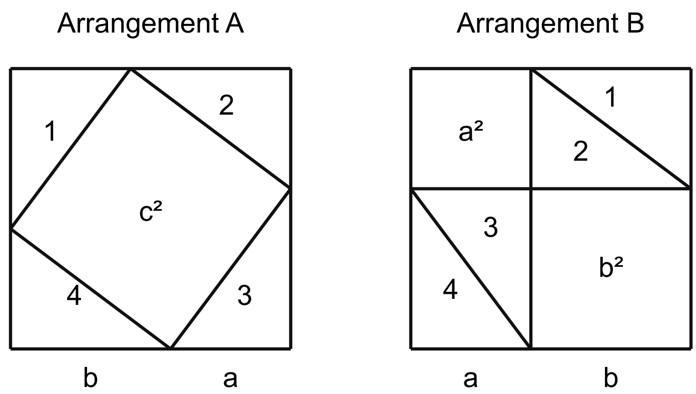



**3** A different cylinder holds 72 cm³. A cone with the same base and height fills it in three equal full pours. Complete the table and explain why dividing by three gives one cone volume.

**DATA TABLE**

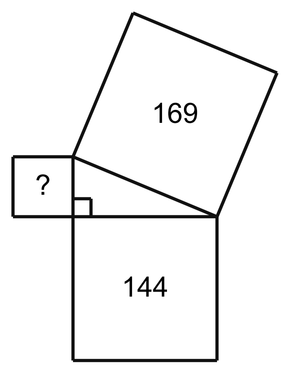
| Full cone pours | Cylinder contents (cm³) |
|---|---|
| 0 | 0 |
| 1 | ___ |
| 2 | ___ |
| 3 | ___ |

**4** Both outer squares have side a + b. The numbered pieces are the same four congruent right triangles with legs a and b and hypotenuse c. Explain why the remaining areas are equal, then derive the Pythagorean theorem.




## APPLY IT
Show your work

**1** The diagram gives areas of squares on a right triangle. Find the missing square area and its side length. Use the area model to explain why you subtract rather than add.




Square areas in cm²

**2** A cylinder has $B = 12$ cm² and $h = 3$ cm. A cone has the same base but $h = 9$ cm. Find both volumes. Explain why the cone is not one third of this cylinder, using the conditions in the pouring model.

## CHALLENGE One step up

Correct both claims using a model on this sheet: "Every cone is one third of every cylinder" and "$a^2 + b^2 = c^2$ works for any triangle." Name the required conditions and explain what the models establish.

# Volume and Triangle Lab



Accept equivalent correct reasoning. Check plotted points and constructed representations as well as written answers. Numbered solutions match the student tasks.

### PRACTICE 1

**$B = 9\pi$ cm²; each layer has volume $9\pi$ cm³, so $V = 45\pi$ cm³. Doubling height makes 10 equal layers and doubles volume to $90\pi$ cm³.**

### PRACTICE 2

**Cylinder: $B = 9\pi$ cm² and $V = Bh = 54\pi$ cm³. Each full cone contributes one of three equal portions, so cone $V = (1/3)Bh = 18\pi$ cm³.**

### PRACTICE 3

**Missing volumes: 24, 48 and 72 cm³. Three identical cone volumes total 72 cm³, so one cone holds 72/3 = 24 cm³.**

### PRACTICE 4

**Equal outer areas minus the same four triangle areas leave equal remainders. A leaves $c^2$; B leaves $a^2 + b^2$. Therefore $a^2 + b^2 = c^2$ for a right triangle.**

### APPLY IT 1

Missing area = 169 - 144 = 25 cm²; the side length is 5 cm.
The hypotenuse square equals the sum of the leg squares, so subtract one leg-square area to find the other.

### APPLY IT 2

Both volumes are 36 cm³: cylinder 12 × 3; cone (1/3) × 12 × 9.
Their heights differ, so the congruent-base, equal-height pouring comparison does not apply.

### CHALLENGE

**The cone/cylinder ratio needs congruent bases and equal heights. The square rearrangement uses congruent right triangles, with c opposite the right angle; the theorem is not true for arbitrary triangles. The models explain relationships, not just numerical recipes.**