

Pack and Count

Investigate volume one layer at a time.

GRADE 5 • 5.GSR.8.3

5.GSR.8.3 "Investigate volume of right rectangular prisms by packing them with unit cubes without gaps or overlaps. Then, determine the total volume to solve problems."

NAME _____

DATE _____

Work with a partner and 24 identical unit cubes. Your teacher supplies open-top rectangular boxes A (3 cubes long, 2 wide, 2 high) and B (4 long, 3 wide, 2 high). All inside dimensions are multiples of the actual cube edge. Reuse cubes between tasks. Diagrams show layer plans, not actual-size templates.

The big idea

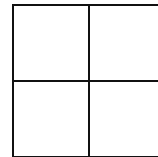
A unit cube has volume 1 cubic unit. Pack cubes edge to edge with no gaps or overlaps. Count cubes in one full layer, then count equal layers. Volume = cubes per layer times number of layers.

Try it together

A small box fits 2 cubes along each bottom edge and 3 layers. How many cubes fill it?

One layer has $2 \times 2 = 4$ cubes. Each square shows one cube position. Three full layers have $4 \times 3 = 12$ cubes. The box holds 12 unit cubes; its volume is 12 cubic units.

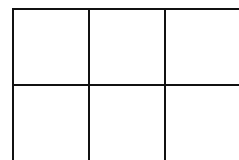
Top view of one full layer



1. Pack the first layer

Place one full layer of cubes in box A with no gaps or overlaps. Mark each position in the plan as you fill it. Record rows, cubes in each row, and cubes in this layer.

Box A: bottom-layer plan



2. Fill the whole box

Add a second full layer to box A. Record cubes per layer, number of layers, and total cubes. Explain why counting only the top layer misses cubes.

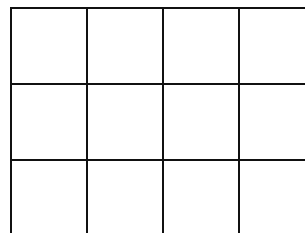
3. Leave a gap

Remove one cube from the top layer of full box A. How many cubes remain? How many more fill the box? Has the volume of the inside of the box changed? Explain.

4. Pack a new box

Empty A. Pack box B one full layer at a time. Mark its layer plan. Record cubes in one layer, full layers, and volume. How many more cubes does B hold than A?

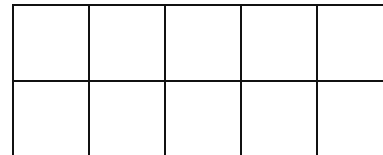
Box B: one-layer plan



5. Packing for delivery

A shipping box fits 5 cubes along its length, 2 along its width, and 2 layers. Build this solid with your cubes. Use the plan to count a layer, then find how many unit-cube packages fill the box.

Shipping box: one-layer plan



6. Enough stock?

A shop must fill three boxes the same size as A. It has 24 unit-cube packages. Use your packing results to decide if it has enough. How many packages are needed or left over?

7. Same cubes, different prisms

Use all 24 cubes to build two different rectangular prisms, one at a time. Draw each bottom-layer plan and label its number of layers. Explain why their volumes are equal.

Teacher notes

Before class, supply 24 identical cubes per pair and two open-top boxes whose inside dimensions fit the specified cubes exactly. Make or choose boxes to match your cubes; do not measure the printed plans. Check actual no-gap packing in tasks 1-4 and the built model in task 5. Each square on a plan represents one cube position, not one cubic unit by itself. Count all full layers and accept rotated arrangements.

Task 1

2 rows of 3 cubes (or 3 rows of 2): 6 cubes in one layer.

Task 2

6 cubes per layer $\times$ 2 layers = 12 cubes; volume 12 cubic units. Six cubes are hidden below the top layer.

Task 3

11 cubes remain; 1 more fills it. The box still has volume 12 cubic units; a gap is unfilled space.

Task 4

12 cubes per layer $\times$ 2 layers = 24 cubic units. B holds 12 more cubes than A.

Task 5

$5 \times 2 = 10$ per layer; $10 \times 2 = 20$ packages fill the box. Volume is 20 cubic units.

Count all cubes in both equal layers, including hidden cubes.

Task 6

Three boxes need $3 \times 12 = 36$ packages. The shop is 12 packages short.

Use the measured 12-cube capacity of A for each of the three boxes.

Task 7

Example: a 4-by-3 layer with 2 layers, and a 6-by-2 layer with 2 layers. Each has 24 cubes, so both volumes are 24 cubic units. Other different dimension sets with product 24 are valid.