

Triangle Proof Power

Explain why triangle theorems hold, beyond a single example.

HIGH SCHOOL • HSG-CO.C.10

CCSS.Math.Content.HSG-CO.C.10 "Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent..."

NAME _____

DATE _____

Materials and setup

Pencil and ruler. Diagrams are reasoning aids; do not measure them as proof. Use words, equations and named reasons.

FROM EVIDENCE TO PROOF

A proof uses definitions and established facts to justify a claim for every allowed case. A measured example can suggest a theorem, but cannot prove it. Tools here: SSS congruence, corresponding parts of congruent triangles, the midpoint formula, parallel-line angle facts and a straight angle of 180° .

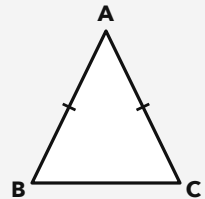
WORKED EXAMPLE

Prove the base angles of the marked isosceles triangle are congruent. Given $AB = AC$, compare triangle ABC with triangle ACB.

1. $AB = AC$ is given; $AC = AB$ is the same equality reversed. $BC = CB$ because it is the same segment.
2. The triangles are congruent by SSS, with A matching A, B matching C and C matching B.
3. Corresponding angles ABC and ACB are congruent. These are the base angles.

Answer:

The base angles are congruent by SSS and corresponding parts.



PRACTICE 4 problems

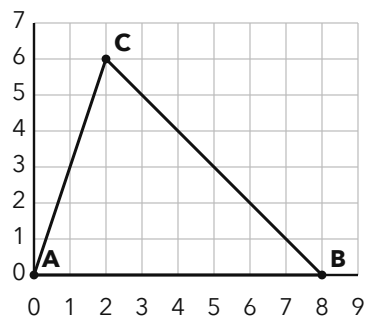
- 1** A student measures three triangles and gets an angle sum close to 180° each time. Does that prove the theorem for all triangles? Explain what is missing.

ANSWER _____

- 2** In the worked proof, why is $BC = CB$ a valid statement, and which angle corresponds to angle ABC?

ANSWER _____

- 3** Use $A(0,0)$, $B(8,0)$, $C(2,6)$ on the supplied grid. Mark D halfway along AC and E halfway along BC; draw DE. Find D, E, DE and AB. Why is this still only an example?



ANSWER _____

- 4** For the general proof, use $A(0,0)$, $B(2a,0)$, $C(2b,2c)$, with $a > 0$ and $c > 0$. D and E are the midpoints of AC and BC. Find their coordinates using the midpoint formula.

ANSWER _____

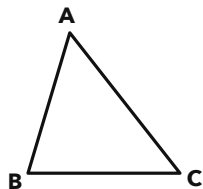
APPLY IT Show your work

- 1** Prove the midpoint-segment theorem for the general triangle in Practice 4. Use D and E to show both that DE is parallel to AB and that DE is half as long as AB. Give reasons; do not cite the theorem you are proving.

ANSWER _____

2

Prove the triangle angle-sum theorem. On the supplied triangle, draw a line $X-A-Y$ through A parallel to BC , with X left of A and Y right of A . Identify the two angles at A equal to the base angles. Use the straight angle at A to finish the proof.



ANSWER _____

CHALLENGE

One step up

A student objects: Your midpoint proof only works when C is directly above A . Use the variable b to evaluate this objection. Explain why $a > 0$ and $c > 0$ are included.

PRACTICE 1

A student measures three triangles and gets an angle sum close to 180° each time. Does that prove the theorem for all triangles? Explain what is missing.

No. Measurements support a conjecture but cover only three examples and may be imprecise. A proof must justify the result for every triangle.

PRACTICE 2

In the worked proof, why is $BC = CB$ a valid statement, and which angle corresponds to angle ABC?

BC and CB name the same segment (reflexive equality). Angle ABC corresponds to angle ACB under the stated vertex matching.

PRACTICE 3

Use $A(0,0)$, $B(8,0)$, $C(2,6)$ on the supplied grid. Mark D halfway along AC and E halfway along BC; draw DE. Find D, E, DE and AB. Why is this still only an example?

$D(1,3)$, $E(5,3)$; $DE = 4$ and $AB = 8$. DE is horizontal like AB. These numbers verify one triangle, not every triangle.

PRACTICE 4

For the general proof, use $A(0,0)$, $B(2a,0)$, $C(2b,2c)$, with $a > 0$ and $c > 0$. D and E are the midpoints of AC and BC. Find their coordinates using the midpoint formula.

$D = ((0+2b)/2, (0+2c)/2) = (b,c)$. $E = ((2a+2b)/2, (0+2c)/2) = (a+b,c)$.

APPLY IT 1

$D(b,c)$ and $E(a+b,c)$ have equal y-coordinates, so DE is horizontal. AB is horizontal at $y = 0$; $c > 0$ makes the lines distinct, hence parallel. $DE = (a+b)-b = a$, while $AB = 2a$. Therefore $DE = AB/2$. Every nondegenerate triangle can be placed with one side on the horizontal axis, its first endpoint at the origin and the third vertex above the axis. Variables allow any $a > 0$, $c > 0$ and real b , so the argument is general, not restricted to the numerical example.

APPLY IT 2

Angle XAB equals angle ABC and angle CAY equals angle ACB by alternate interior angles. Angles XAB, BAC and CAY form straight angle XAY, so their measures sum to 180° . Substituting gives angle ABC + angle BAC + angle ACB = 180° .

The proof uses a parallel through a vertex and established parallel-line/straight-angle facts, not a measured drawing or the triangle angle-sum theorem itself. Keep X, A and Y on the same straight line; angles XAB and CAY are outside the triangle at A.

CHALLENGE

The objection is false. C has x-coordinate $2b$, and b can be any real number; it need not be zero.

Choosing $a > 0$ gives AB positive length.

Choosing $c > 0$ keeps C off the base line, so the triangle is nondegenerate. We may rotate/translate a triangle and select orientation to use this setup.

Require a connected argument with valid reasons for each proof. A correct numerical answer or diagram alone is not a proof. Practice 3 is deliberately a numerical illustration before the general proof. The midpoint proof establishes the theorem for the segment joining the midpoints of AC and BC; renaming vertices covers the other side pairs. Accept equivalent synthetic or coordinate proofs with stated givens and no circular use of the target theorem. The worked isosceles proof compares the same triangle under a vertex swap; SSS supports that congruence correspondence. This sheet focuses on these theorems rather than claiming to prove every listed triangle theorem.