Why the Squares Add

Explain the theorem through areas and rearrangements.

GRADE 8 · TEKS 8.6.C

8.6.C "use models and diagrams to explain the Pythagorean theorem."

NAME ______________________ DATE ____________

CONNECT THE REPRESENTATIONS

For a right triangle, let a and b be the legs and c the hypotenuse. The squares built on these sides have areas a^2, b^2 and c^2. An area model explains the relationship; substitution alone checks only one example.

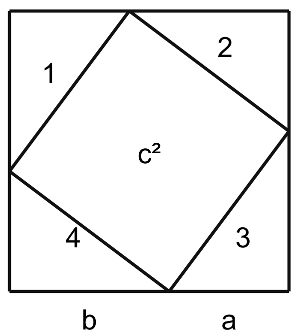
WORKED EXAMPLE

Both outer squares have side $a + b$. Each contains the same four right triangles. Use the remaining areas to explain $a^2 + b^2 = c^2$.

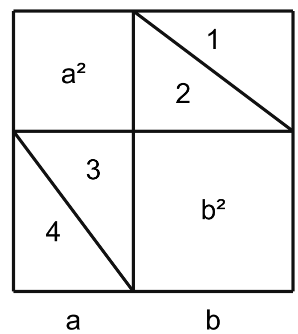
1. In A, the four hypotenuses enclose a square of area c^2. Adjacent acute angles total 90°, making its corners right angles.
2. In B, the uncovered regions are squares of sides a and b. Rearrangement preserves all four triangle areas.

Answer: Equal whole areas minus the same four triangle areas leave equal remainders: $a^2 + b^2 = c^2$.

Arrangement A



Arrangement B



PRACTICE 4 problems

1 In the worked model let $a = 3$ and $b = 4$. Calculate the outer-square area, the area of all four triangles, and the uncovered area. How does each arrangement explain that remainder?

____

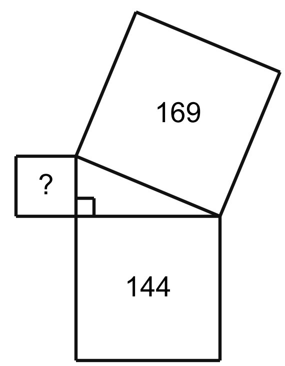
____

____

____

2 A student says the middle shape in arrangement A is a square just because its sides look equal. Explain why both its sides and angles justify calling it a square.

3 The labels inside these attached squares are areas in cm². Find the missing area and the corresponding leg length. Explain why you subtract square areas rather than side lengths.



Square areas in cm²

4 In a new right triangle the leg-square areas are 36 and 64 cm². Sketch a right triangle with a square on each side and label their areas. Explain the hypotenuse-square area and side length.

APPLY IT

Show your work

1 A right-triangle mural has area 180 cm² and one leg 15 cm. Find the other leg and the exact hypotenuse. Explain the area of the square on each side, and test the claim that the hypotenuse is 29 cm.

2 A student doubles both legs of the worked 3-4-5 triangle. Compare its new leg-square areas and hypotenuse-square area with the originals. Explain why the hypotenuse doubles rather than quadruples.

CHALLENGE One step up

A triangle has side lengths 4, 5 and 6. Can its three attached square areas satisfy the same relationship? Use the largest square to explain why the right-triangle condition cannot be omitted.

Why the Squares Add

Accept equivalent area models and reasoning. Numbered solutions match the student tasks.

PRACTICE 1

Outer area 49; each triangle has area 6, so four occupy 24. The remainder is 25: A gives c^2, while B gives 9 + 16. Thus c = 5.

PRACTICE 2

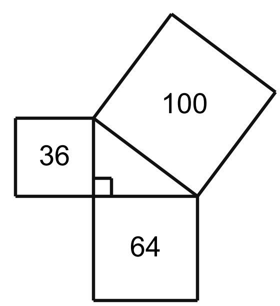
Four congruent hypotenuses give equal sides c. At each central corner, neighboring acute angles total 90°, leaving a 90° angle. Equal sides alone would not prove a square.

PRACTICE 3

Missing area: 169 - 144 = 25 cm^2; leg: 5 cm. Subtract square areas, not the side lengths 13 - 12.

PRACTICE 4

Leg-square areas: 36 and 64. The hypotenuse square has area 100 cm^2 and side 10 cm; it sits opposite the right angle, as shown.



APPLY IT 1

The other leg is 24 cm; the hypotenuse is $\sqrt{801} = 3\sqrt{89}$ cm.
From (15b)/2 = 180, b = 24. The leg-square areas are 225 and 576, totaling 801. Since $29^2 = 841$, 29 cm is incorrect.

APPLY IT 2

New square areas are 36, 64 and 100 cm^2.
Each area is four times its original 9, 16 or 25; taking square roots makes side lengths twice as large, so the hypotenuse becomes 10 cm.

CHALLENGE

No: $4^2 + 5^2 = 41$, while $6^2 = 36$. The longest side is 6, so no relabeling makes the largest square equal the other two combined. The right-angle premise is essential.