LESSON MAKER PRO

Balance, Compare, Explain

Connect algebraic models with geometric arguments.

GRADE 8 · TEKS 8.8

8.8 "Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations..."

NAME ______________________________ DATE ______________

CONNECT THE REPRESENTATIONS

An equation asks when quantities match; an inequality compares them. State what the variable and coefficients mean, and preserve equivalence when solving. In geometry, parallel-line angle relationships support informal arguments for triangle angle sums, exterior angles and similarity.

WORKED EXAMPLE

A rental costs (3/2)x + 6 dollars; another costs (1/2)x + 12 dollars for x hours. Model equal costs and solve, preserving both sides at each step.

DATA TABLE

Equal-cost model	Equivalent step
(3/2)x + 6 = (1/2)x + 12	Subtract (1/2)x
x + 6 = 12	Subtract 6
x = 6	Check both original costs

1. The rates are dollars per hour and the constants are starting fees.
2. Subtract equal quantities from both sides: x = 6. Check (3/2)(6) + 6 = 15 and (1/2)(6) + 12 = 15.

Answer: They match at 6 hours, costing $15 each.

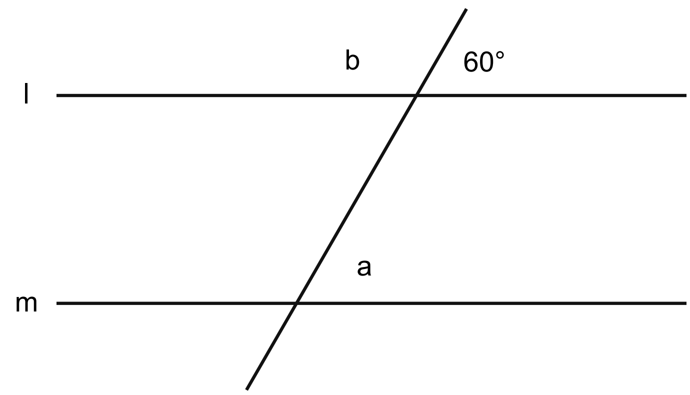
PRACTICE 4 problems

1. Plan A pays a club \$0.75 per ticket plus \$8; Plan B pays \$0.50 per ticket plus \$14. Write and solve an equation with x on both sides for equal earnings. Check the ticket count.

2. Write two different real-world questions: one modeled by $(3/2)x + 4 = (1/2)x + 10$, and one by $(1/2)x + 6 \le (3/2)x + 2$. Define x and the units, then solve each.

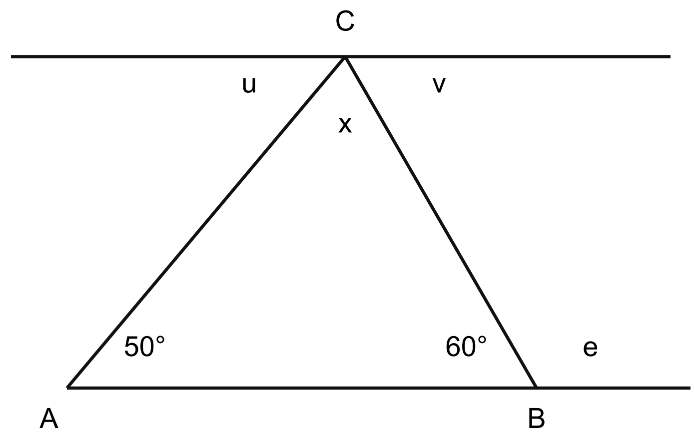
3. For x hours, Rental A costs \$1.25 per hour plus \$5 and B costs \$0.75 per hour plus \$11. Write and solve an inequality showing when A costs no more than B. State a realistic domain.

4 Lines l and m are parallel. Find a and b. Explain corresponding angles and the straight-angle relationship; justify why continuing the same direction through both parallel lines preserves the corresponding angle.

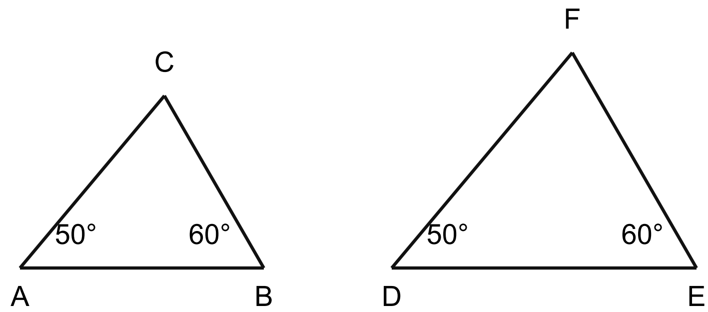


APPLY IT
Show your work

1 The line through C is parallel to AB. Use alternate interior angles to find u and v; then explain the triangle angle sum and the exterior-angle rule by finding x and e.



2 Use the two triangles to explain the angle-angle similarity criterion. Find the third angles and give the vertex correspondence. Must the triangles be congruent? Explain.



CHALLENGE One step up

Compare $2(x + 3) = 2x + 6$ and $2(x + 3) = 2x + 8$. A student claims both have $x = 0$ after canceling $2x$. Explain what the remaining statements mean and give each solution set.

Balance, Compare, Explain

Accept equivalent correct reasoning. Check plotted points and constructed representations as well as written answers. Numbered solutions match the student tasks.

PRACTICE 1

$0.75x + 8 = 0.50x + 14$ gives $0.25x = 6$ and $x = 24$ tickets. Both totals are \$26.

PRACTICE 2

Example: compare two hourly rentals costing \$1.50/h plus \$4 and \$0.50/h plus \$10; equal at $x = 6$ hours. Another pair costs \$0.50/h plus \$6 and \$1.50/h plus \$2; ask when the first costs no more than the second. Its solution is $x \geq 4$ hours. Accept other consistent contexts.

PRACTICE 3

$1.25x + 5 \leq 0.75x + 11$ gives $0.50x \leq 6$, so $x \leq 12$. With nonnegative rental time, $0 \leq x \leq 12$ hours; integer-only hours are acceptable if explicitly stated.

PRACTICE 4

$a = 60°$ by corresponding angles: a translation taking one parallel line to the other keeps its direction and its angle with the transversal. $b = 120°$ because b and 60° form a straight angle totaling 180°.

APPLY IT 1

$u = 50°$, $v = 60°$, $x = 70°$ and $e = 120°$.
The angles u, x and v form a straight angle, so $x + 50 + 60 = 180$. At B, $e + 60 = 180$, hence $e = 120 = 50 + 70$, the remote interior-angle sum.

APPLY IT 2

Both third angles are 70°, so $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$.
Matching two angles forces the third to match; resizing one side to its partner makes the corresponding rays meet at the same third vertex, giving the same shape. They need not have equal side lengths or be congruent.

CHALLENGE

The first becomes $6 = 6$, true for every real x. The second becomes $6 = 8$, false for every x, so it has no solution. Canceling x does not imply $x = 0$.