

Triangle Distance Lab

Choose a rule, solve a triangle, and justify your result.

HIGH SCHOOL • HSG-SRT.D

CCSS.Math.Content.HSG-SRT.D "Apply trigonometry to general triangles"

NAME _____

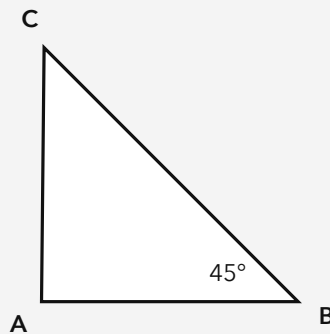
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CHOOSE A TRIANGLE RULE

Let side a face angle A , b face B , and c face C . Use $c^2 = a^2 + b^2 - 2ab \cos C$ for SAS or SSS. Use $a/\sin A = b/\sin B = c/\sin C$ when a known angle has a known opposite side. Area = $\frac{1}{2}ab \sin C$ when C is the included angle. SSA may have zero, one, or two triangles. Work in degrees. Show a formula and substitution; give exact values unless asked to round.

WORKED EXAMPLE

$AB = 5$, $BC = 7$, and angle $B = 45^\circ$. Find AC exactly. The diagram uses these actual proportions.

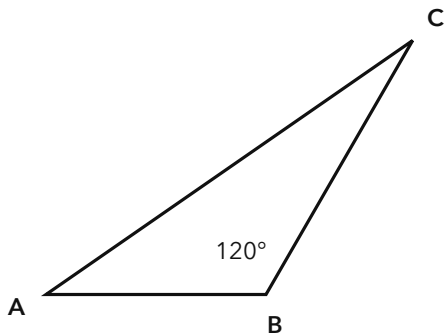


1. Two known sides meet at the known angle, so use the cosine rule.
2. $AC^2 = 5^2 + 7^2 - 2(5)(7) \cos 45^\circ$.
3. $AC^2 = 74 - 35\sqrt{2}$.
4. Lengths are positive: take the positive square root.

Answer: $AC = \sqrt{74 - 35\sqrt{2}} \approx 4.95$.

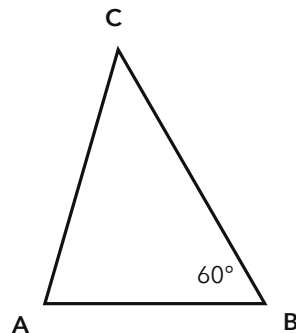
PRACTICE 6 problems

- 1** $AB = 3$, $BC = 4$, and angle $B = 120^\circ$. Find AC exactly; show your substitution.



ANSWER _____

- 2** $AB = 6$, $BC = 8$, and angle $B = 60^\circ$. Find AC exactly; show your substitution.



ANSWER _____

- 3** Two sides measure 8 cm and 10 cm with included angle 150° . Find the triangle's exact area.

ANSWER _____

- 4** A triangle has $a = 7$, $b = 8$, and $c = 9$. Find angle C to the nearest tenth of a degree.

ANSWER _____

- 5** In triangle ABC, $A = 30^\circ$, $B = 105^\circ$, and $a = 6$. Find b to the nearest tenth and angle C exactly.

ANSWER _____

- 6** Can a triangle have $A = 30^\circ$, $a = 3$, and $b = 8$? Use the sine rule to justify your answer.

ANSWER _____

APPLY IT Show your work

- 1** Two straight paths from a park entrance form a 120° angle. Students walk 12 m and 16 m along the two paths. Find their exact separation and the exact area of the triangle formed by their positions and the entrance. Name why each rule applies.

ANSWER _____

2

Derive the area rule for an acute triangle ABC. Let $AB = c$ and $AC = b$. Draw altitude CD to AB; call its height h . Express h using b and angle A, then substitute into the base-height area formula to obtain $\text{area} = \frac{1}{2}bc \sin A$. Explain why the same height expression works when A is obtuse and D lies beyond A on line AB.

ANSWER _____

CHALLENGE One step up

In triangle ABC, $A = 30^\circ$, $a = 6$, and $b = 6\sqrt{3}$. Find every possible B, C, and c. Explain why both cases work and why there are no other triangles.

PRACTICE 1

$$AC^2 = 9 + 16 - 24 \cos 120^\circ = 37; AC = \sqrt{37}.$$

PRACTICE 2

$$AC^2 = 36 + 64 - 96 \cos 60^\circ = 52; AC = 2\sqrt{13}.$$

PRACTICE 3

$$\text{Area} = \frac{1}{2}(8)(10) \sin 150^\circ = 20 \text{ cm}^2.$$

PRACTICE 4

$$\cos C = (49 + 64 - 81)/(2 \cdot 7 \cdot 8) = 2/7; C \approx 73.4^\circ.$$

PRACTICE 5

$$C = 45^\circ; b = 6 \sin 105^\circ / \sin 30^\circ \approx 11.6.$$

PRACTICE 6

$$\sin B = 8 \sin 30^\circ / 3 = 4/3, \text{ which exceeds } 1. \text{ No such triangle exists.}$$

APPLY IT 1

$$\text{Separation} = 4\sqrt{37} \text{ m}; \text{ area} = 48\sqrt{3} \text{ m}^2.$$

SAS gives $\text{distance}^2 = 12^2 + 16^2 - 2(12)(16) \cos 120^\circ = 592$. The included angle also gives $\text{area} = \frac{1}{2}(12)(16) \sin 120^\circ = 48\sqrt{3}$. Use positive lengths and square units for area.

APPLY IT 2

$$h = b \sin A, \text{ so area} = \frac{1}{2}ch = \frac{1}{2}bc \sin A.$$

In right triangle ACD, $\sin A = h/b$ for acute A. For obtuse A, the exterior acute angle is $180^\circ - A$; $h = b \sin(180^\circ - A) = b \sin A$, so the area formula still works. AB is the base in both cases.

CHALLENGE

$\sin B = (6\sqrt{3} \sin 30^\circ)/6 = \sqrt{3}/2$. In $0^\circ < B < 150^\circ$, the only possibilities are $B = 60^\circ$ or 120° . If $B = 60^\circ$, $C = 90^\circ$ and $c = 12$. If $B = 120^\circ$, $C = 30^\circ$ and $c = 6$. Both have positive angles summing to 180° and the required sides.

A calculator in degree mode is needed. Accept equivalent radical expressions. Require a rule justified by the givens, work for both SSA cases, and units where specified. In applied problem 2, students sketch their own altitude; the derivation must cover the exterior-foot case for an obtuse angle.