

Angle Connection Lab

Prove line and angle relationships with clear reasons.

HIGH SCHOOL • HSG-CO.C.9

CCSS.Math.Content.HSG-CO.C.9 "Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent..."

NAME _____

DATE _____

Materials and setup

Pencil and straightedge. Write paragraph proofs or statement-and-reason chains. You may use rigid motions, the unique parallel through a point, linear pairs, and SAS/SSS congruence. Do not cite the theorem a task asks you to prove. Lines in a drawing extend in both directions.

THE BIG IDEA

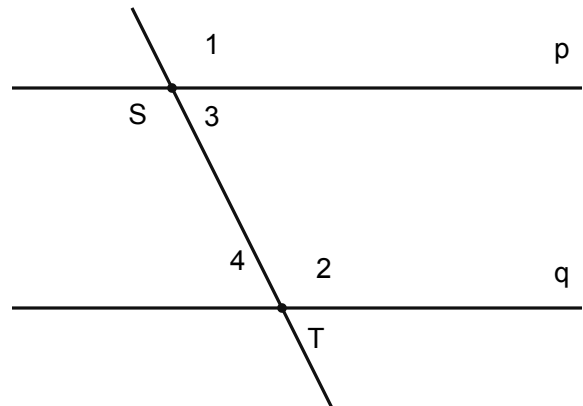
A proof explains why a relationship always follows from its givens. Rigid motions preserve angle measures. Parallel lines have the same direction. Linear pairs sum to 180° . To prove an "if and only if" statement, prove both directions. A diagram suggests facts; use the stated givens as your premises.

WORKED EXAMPLE

Given p parallel to q , prove corresponding angles 1 and 2 are congruent by a translation.

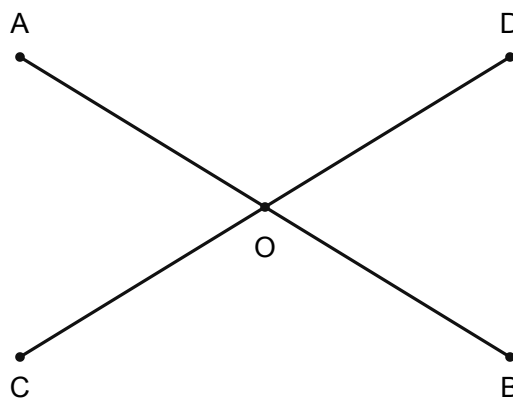
1. Translate S to T along the transversal. The transversal maps onto itself.
2. The image of p passes through T and is parallel to p , so it is q by the unique-parallel property.
3. The two rays forming angle 1 map to the corresponding rays of angle 2. Translation preserves their angle measure.

Answer: Therefore angle 1 is congruent to angle 2.



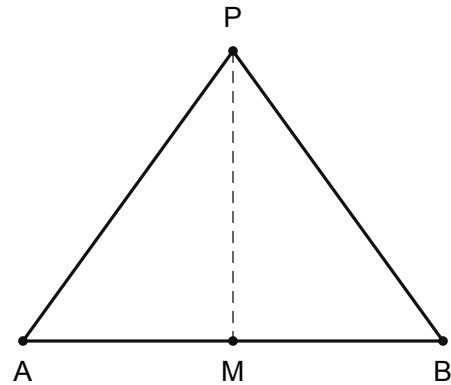
PRACTICE 4 problems

- 1** Lines AB and CD meet at O. Prove angle AOC is congruent to angle BOD using linear pairs.



- 2** In the worked-example diagram, p is parallel to q. Use the worked result about angles 1 and 2 and a linear-pair argument to prove alternate interior angles 3 and 4 are congruent.

- 3** M is the midpoint of AB, and PM is perpendicular to AB. Prove $PA=PB$. Explain which triangle-congruence rule applies.



- 4** Use the diagram from Practice 3, with these new givens. Now assume $PA=PB$ and M is the midpoint of AB, with P not on AB. Prove PM is perpendicular to AB. Do not assume the perpendicular-bisector theorem.

APPLY IT Show your work

- 1** A support point P must be equally far from anchors A and B. Describe all possible locations of P and justify both directions of your answer. Include the case $P=M$, the midpoint.

- 2** Two parallel stripes are crossed by a diagonal. A corresponding angle pair is 118° and $(7x-8)^\circ$. Find x . Explain why solving the equation uses an established theorem rather than proving it.

CHALLENGE One step up

A student measures one vertical-angle pair and says, "They match, so all vertical angles are congruent." Explain the gap. Give a proof that works for every pair, using a shared adjacent angle of measure z .

Require stated givens and reasons; measurements alone do not prove a theorem. Practice 2 may use corresponding angles plus equal supplements or vertical angles. Practice 3 and 4 establish both directions of the perpendicular-bisector theorem; accept valid alternative proofs without circular reasoning.

PRACTICE 1

Angles AOC and AOD sum to 180° , as do BOD and AOD. Subtract the common measure of AOD to get $m\angle AOC = m\angle BOD$. Thus vertical angles are congruent.

PRACTICE 2

Angles 1 and 2 are congruent by the translation argument. Angles 2 and 4 are a linear pair, as are 1 and 3. Subtracting equal angle measures from 180° gives $m\angle 3 = m\angle 4$. Equivalently, angle 3 corresponds to the lower-right angle at T, which is vertical to angle 4.

PRACTICE 3

$AM = BM$ because M is the midpoint. Angles PMA and PMB are right angles, and $PM = PM$. Thus triangles PMA and PMB are congruent by SAS. Corresponding sides PA and PB are equal.

PRACTICE 4

$PA = PB$, $AM = BM$ and $PM = PM$ imply triangles PMA and PMB are congruent by SSS. Angles PMA and PMB are equal and form a linear pair, so each is 90° . Thus PM is perpendicular to AB.

APPLY IT 1

The locations form the perpendicular bisector of AB.

A point on that bisector is equidistant from A and B by SAS (or is M itself). Conversely, an equidistant point off AB lies on the bisector by the SSS and linear-pair proof. On AB, only midpoint M is equidistant.

APPLY IT 2

$x = 18$.

The corresponding-angle theorem gives $7x - 8 = 118$. Solving gives $x = 18$. This is one numerical application; the translation argument establishes equality for every such parallel-line configuration.

CHALLENGE

A measurement checks only one drawing and may be imprecise. For any vertical pair with measures a and b, both form linear pairs with the shared adjacent angle z. Thus $a + z = 180^\circ$ and $b + z = 180^\circ$, so $a = b$ after subtraction.