

Angle Match Mission

Explain why two matching angles force triangle similarity.

HIGH SCHOOL • HSG-SRT.A.3

CCSS.Math.Content.HSG-SRT.A.3 "Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar."

NAME _____

DATE _____

Materials and setup

Pencil and straightedge. Use rigid motions, the fact that dilation preserves angles, and the unique intersection of two nonparallel rays. Do not use AA itself as a premise when establishing AA. Triangles are nondegenerate. The drawings illustrate the stated matching angles.

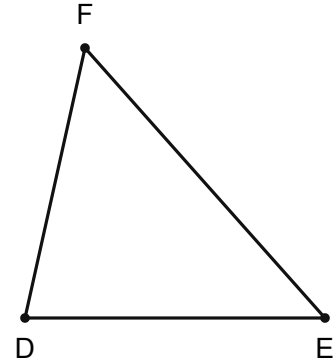
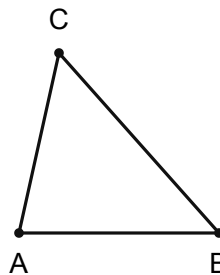
THE BIG IDEA

To establish AA, start with $\angle A = \angle D$ and $\angle B = \angle E$. Rigid motions align A with D and ray AB with ray DE; reflect if needed so C and F lie on the same side. Dilate about D by DE/AB to send B to E. The equal angles force the image of C onto both ray DF and ray EF, whose unique intersection is F. Thus a similarity transformation maps the whole triangle.

WORKED EXAMPLE

For the shown triangles, suppose $\angle A = \angle D$, $\angle B = \angle E$, $AB=4$ and $DE=6$. Describe a transformation that maps triangle ABC onto DEF without citing AA.

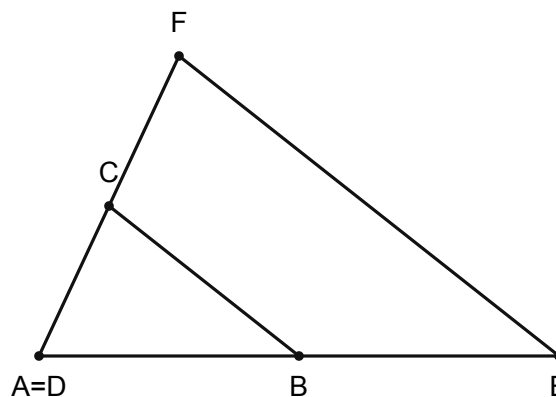
1. Translate and rotate to place A at D and ray AB on ray DE. Reflect across DE if necessary to align the triangles' sides of the line.
2. Dilate about D by $6/4=3/2$. A maps to D and B maps to E.
3. Angle preservation forces the third vertex onto rays DF and EF. Their only intersection is F.



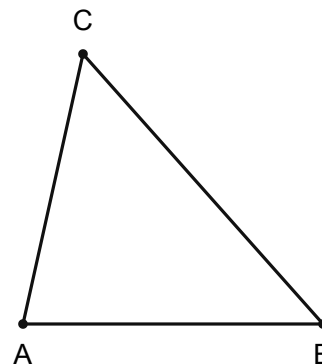
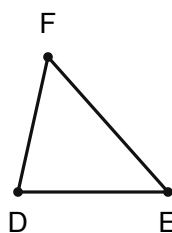
Answer: A rigid motion followed by dilation with $k=3/2$ maps ABC onto DEF.

PRACTICE 4 problems

- 1** The diagram shows aligned triangles with $A=D$, B on ray DE , and C on ray DF . Suppose $DE=2AB$ and the angles at B and E match. Explain why dilation about D with $k=2$ sends C exactly to F .



- 2** In the paired triangles, suppose the two angle pairs match, $AB=10$ and $DE=5$. State the dilation factor and explain why it is DE/AB , rather than AB/DE . Explain how you know the third vertex lands correctly.



- 3** Why may a reflection be needed before dilation? Explain what would go wrong if the aligned triangles were on opposite sides of the shared base line and you used only a positive dilation.

- 4** A student says, "The triangles are similar by AA, so a dilation maps them; therefore AA is true." Explain the circular step and replace it with the key geometric reason that locates the third vertex.

APPLY IT Show your work

- 1** A triangular badge has angles 40° and 65° . A proposed enlarged sign has the corresponding angles 40° and 65° . The corresponding base lengths are 3 cm and 12 cm. Justify that the sign is a scaled copy using the transformation argument, then give k and the third angle.

2

Two triangle designs share one 50° angle. One has another angle of 60° and the other has another angle of 70° . Does this information prove they are not similar? Find all angles, then decide with a valid correspondence.

CHALLENGE One step up

Two triangles each have a 50° angle, but no other information is given. Give angle sets showing why one equal angle is insufficient. Explain which step in the AA transformation argument is no longer forced.

Require a transformation argument when requested, not only the phrase “by AA.” A translation, rotation and optional reflection align the base and side, then the dilation factor is target base/original base. Equal first angles fix one ray; equal second angles fix the other. Drawings show corresponding vertices and orientation; use the stated lengths for scale factors.

PRACTICE 1

Dilation sends B to E and keeps the third vertex on ray DF. It preserves the angle at B, so the ray from E toward the image of C is EF on the same side of DE. Hence the image lies at $DF \cap EF$, which is F.

PRACTICE 2

To map ABC to DEF, $k=5/10=1/2$, so AB becomes DE. After rigid alignment, the equal angles force the third image vertex onto both rays DF and EF, so it is F. The reciprocal 2 maps lengths in the opposite direction.

PRACTICE 3

A positive dilation keeps a point on its ray from the center and on the same side of the base line. Reflecting across that line places the triangles on the same side while preserving angles and lengths, so their angle rays can coincide.

PRACTICE 4

The student assumes AA to prove AA. Instead, align the first vertex and base ray, dilate the base to the target length, and use equal angles to put the third vertex on the two target rays. Their unique intersection is the target third vertex.

APPLY IT 1

$k=4$; both third angles are 75° .

Rigidly align the first vertices and base rays; reflect if needed. Dilate by $12/3=4$. Equal first and second angles locate the third image vertex at the intersection of the sign's corresponding rays. Thus the transformation maps the badge to the sign.

APPLY IT 2

They are similar: both have angles 50° , 60° and 70° .

The missing angles are 70° and 60° . Match $50 \leftrightarrow 50$, $60 \leftrightarrow 60$, $70 \leftrightarrow 70$ even though the order of the given angles differed. The established two-angle transformation argument applies with that correspondence.

CHALLENGE

For example, angles $(50^\circ, 60^\circ, 70^\circ)$ and $(50^\circ, 80^\circ, 50^\circ)$ share 50° but are not similar. Aligning that angle and scaling a base does not fix the second angle ray. The third image vertex need not lie on the target ray from the second vertex, so it need not land at the target third vertex.