

Compound Chance Lab

Choose and justify rules for compound events.

HIGH SCHOOL • HSS-CP.B

CCSS.Math.Content.HSS-CP.B "Use the rules of probability to compute probabilities of compound events in a uniform probability model"

NAME _____

DATE _____

Materials and setup

Pencil. All stated random choices are uniform: each eligible object or selection is equally likely. For counting, $C(n,r)$ counts unordered r -element selections; $P(n,r)$ counts ordered selections without repetition. Show the rule and substitution, not only a final fraction.

THE BIG IDEA

For OR, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. For AND, $P(A \text{ and } B) = P(A)P(B|A)$, where $P(B|A)$ is the probability of B after restricting to A . Only for independent events can $P(B|A)$ be replaced by $P(B)$. Also $P(A|B) = P(A \text{ and } B)/P(B)$, when $P(B) > 0$. Without replacement, update both the eligible counts and the total. Use combinations when order does not matter and permutations when it does.

WORKED EXAMPLE

Of 20 students, 12 play a sport, 9 play an instrument, and 5 do both. Find the chance a uniformly chosen student does at least one.

1. Adding $12+9$ counts the 5 students in both groups twice.
2. Subtract the overlap once: $12+9-5=16$.
3. $P(\text{sport or instrument}) = 16/20 = 4/5$.

Answer: 4/5; 16 of the 20 students do at least one.

PRACTICE 4 problems

- 1** An independent fair coin flip and fair six-sided die roll occur. Find $P(\text{tails and even})$. Identify why you may multiply the two unconditional probabilities.

ANSWER _____

- 2 An integer is chosen uniformly from 1 through 11. Find the probability it is divisible by 2 or by 3. Show the overlap you subtract.

ANSWER _____

- 3 A bag has 7 gold and 10 silver wristbands. Draw two uniformly without replacement. Find $P(\text{first gold and second silver})$. State the conditional probability used.

ANSWER _____

- 4 From the table, a student is chosen uniformly from the students who take art. Find the probability that student also takes music. Explain which group is now the sample space.

CLASSES TAKEN

	Music	No music	Total
Art	5	7	12
No art	4	4	8
Total	9	11	20

ANSWER _____

APPLY IT Show your work

- 1** A playlist has 11 festival-artist songs and 12 nonfestival songs. Two different songs are shuffled uniformly into first and second position. Find the probability that the first is a festival song and the second is a nonfestival song. Explain the changed denominator.

ANSWER _____

- 2** A club has 6 distinct members. It selects a chair and a secretary uniformly without replacement. Find the probability that Ana is chair and Ben is secretary, using permutations. Why would $C(6,2)$ alone be the wrong denominator for this ordered result?

ANSWER _____

CHALLENGE One step up

A council has 7 juniors and 10 seniors. Three members are selected uniformly without replacement. Use combinations to find the probability of exactly 2 juniors. Then check by counting the three possible senior positions in an ordered draw.

PRACTICE 1

$(1/2)(3/6)=1/4$. Independence makes $P(\text{even}|\text{tails})=P(\text{even})=1/2$.

PRACTICE 2

Multiples of 2: 5; multiples of 3: 3; overlap {6}: 1. Thus $(5+3-1)/11=7/11$.

PRACTICE 3

$P(\text{silver second}|\text{gold first})=10/16$. The compound probability is $(7/17)(10/16)=35/136$.

PRACTICE 4

There are 12 art students, of whom 5 also take music. $P(\text{music}|\text{art})=5/12$; the sample space is the art group.

APPLY IT 1

$(11/23)(12/22)=6/23$.

After a festival song is first, 22 songs remain, including all 12 nonfestival songs. The second probability is conditional on the first selection.

APPLY IT 2

$P(6,2)=6 \times 5=30$ equally likely ordered assignments, with 1 favorable assignment, so the probability is $1/30$.

$C(6,2)=15$ counts unordered pairs and merges Ana-chair/Ben-secretary with the reverse assignment. The roles are distinct, so order matters.

CHALLENGE

$C(7,2)C(10,1)/C(17,3)=210/680=21/68$. Each order JJS, JSJ or SJJ has probability $7/68$, so their sum is $21/68$.

Accept equivalent exact probabilities with the model and reasoning shown. Conditional samples must use the restricted group as denominator. Independence is explicitly stated for the coin/die task; do not infer it for draws without replacement. The art/music table counts 20 distinct students exactly once in its four interior cells.