

Polynomial Pathways

Use zeros, multiplicity, and end behavior to sketch graphs.

HIGH SCHOOL • MA.912.AR.6.5

MA.912.AR.6.5 "Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior."

NAME _____

DATE _____

ZEROS, MULTIPLICITY AND SHAPE

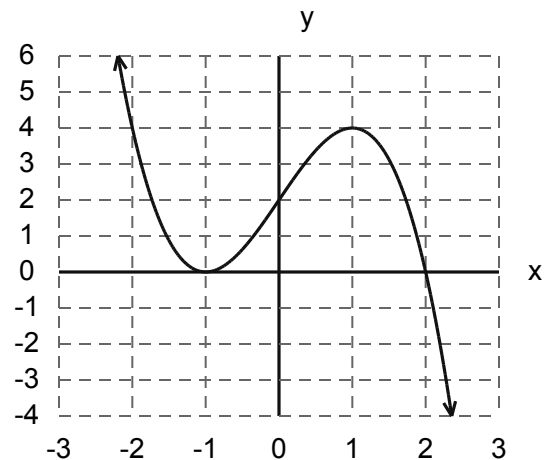
Use a factored polynomial to build a rough graph. A zero of even multiplicity touches the x-axis; a zero of odd multiplicity crosses. Higher multiplicity makes the graph flatter near the zero. Degree and leading coefficient determine end behavior. Plot the y-intercept and extra points to place the curve. A viewing window does not restrict the domain.

WORKED EXAMPLE

Sketch $f(x) = -(x+1)^2(x-2)$. Explain each zero and both ends.

1. Degree 3 and negative leading coefficient: left end rises, right end falls.
2. At $x = -1$ the multiplicity is 2, so the graph touches. At $x = 2$ it crosses.
3. Plot $(0, 2)$, then draw a smooth curve through the required features.

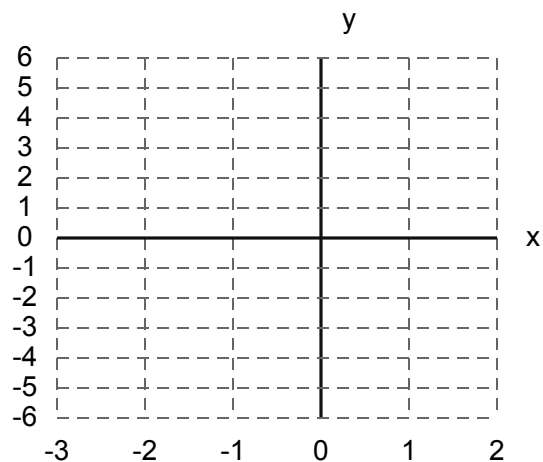
Answer: Touch $(-1, 0)$; cross $(2, 0)$; y-intercept $(0, 2)$; left up, right down.



PRACTICE 4 problems

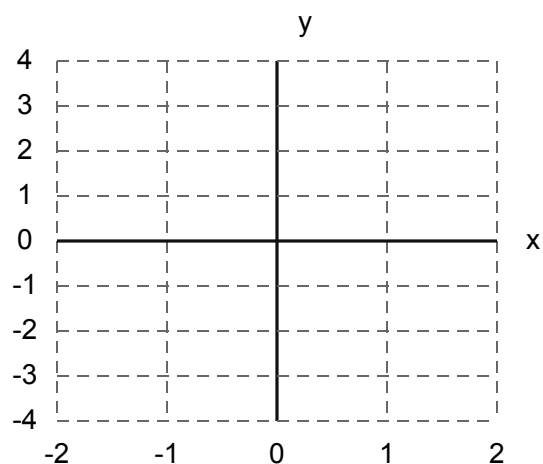
- 1** Sketch $f(x)=(x+2)^2(x-1)$. Label the zeros and y-intercept; identify touch/cross behavior and both ends.

ANSWER _____



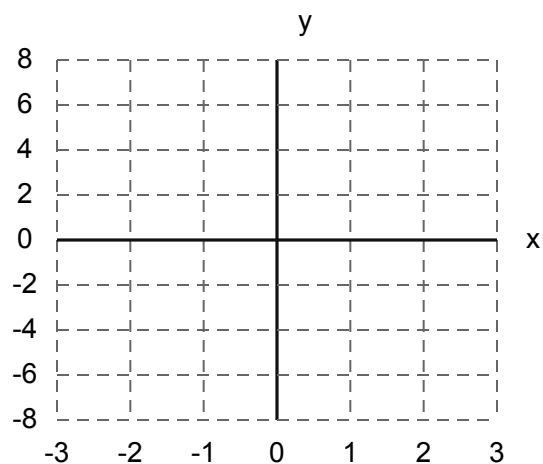
- 2** Sketch $f(x)=-(x-1)^3(x+1)^2$. Explain how the graph behaves near each zero and at both ends.

ANSWER _____



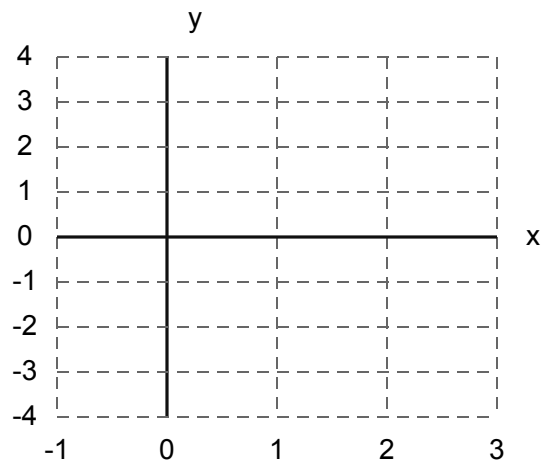
- 3** Sketch $f(x)=(1/4)(x+2)x(x-2)^2$. Identify zeros, multiplicities, end behavior and intervals where f is negative.

ANSWER _____



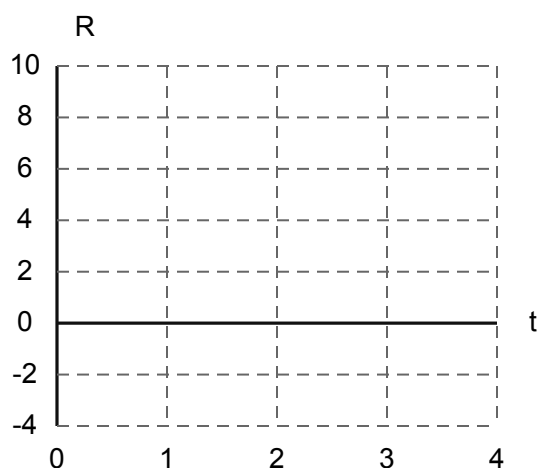
- 4 Sketch $f(x)=(x-1)^5$. How does its multiplicity affect the crossing? Give the degree and both ends.

ANSWER _____



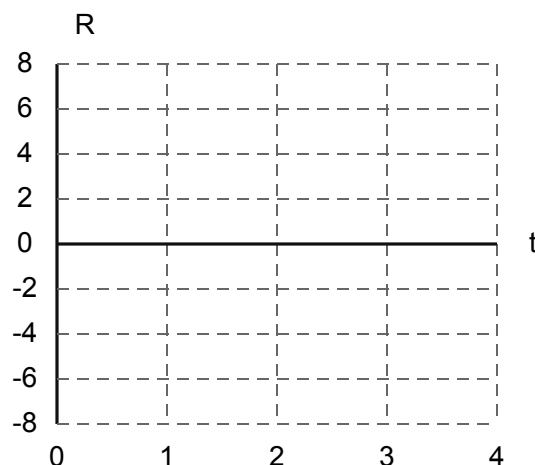
APPLY IT Show your work

- 1 A sensor records signed response $R(t)=(t-1)^2(t-3)$ for $0 \leq t \leq 4$ seconds. Sketch that interval with closed endpoints. At which times does the response touch or cross zero?



2

A simulation gives signed response $R(t) = -(t-2)^3$ for $0 \leq t \leq 4$ seconds. Sketch the recorded interval. Describe the zero and how the response changes through it.

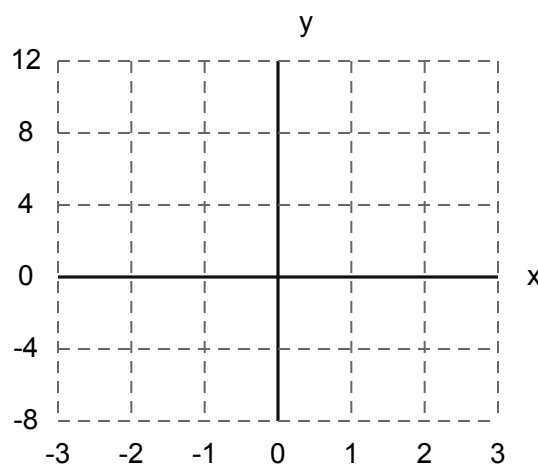


CHALLENGE

One step up

Sketch $f(x) = -(1/4)x^2(x-2)^3(x+2)$. Label each zero and its behavior, both ends, and all intervals where f is positive.

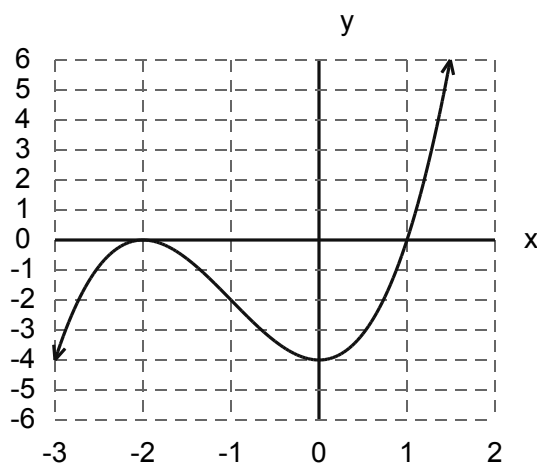
WORK SPACE AND ANSWER



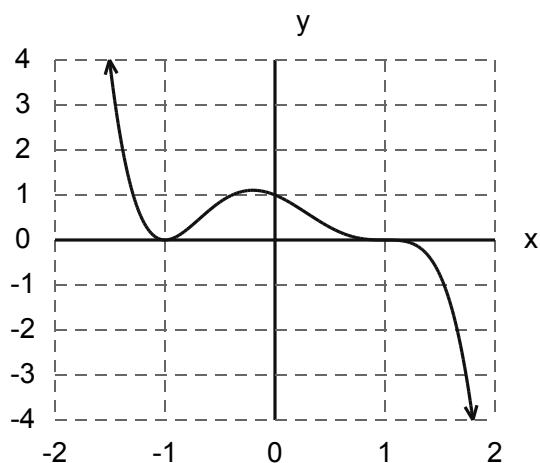
Require an actual sketch with intercepts, touch/cross behavior and continuation arrows. Turning points need only be qualitatively placed. For recorded intervals, use closed endpoints instead of continuing beyond the stated time. Zeros are excluded from strict positive/negative intervals.

PRACTICE 1

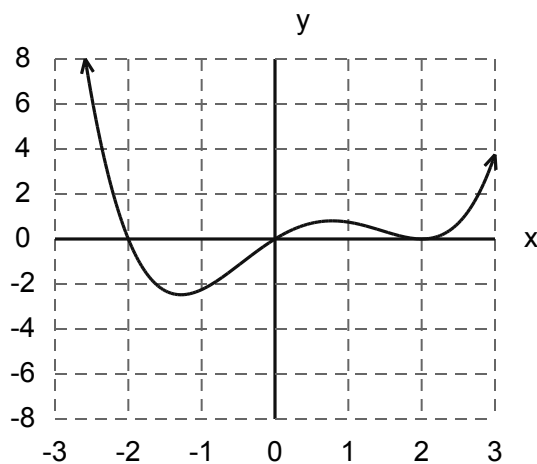
**Touch $(-2,0)$, cross $(1,0)$; y-intercept $(0,-4)$.
Degree 3, positive leading coefficient: left down, right up.**

**PRACTICE 2**

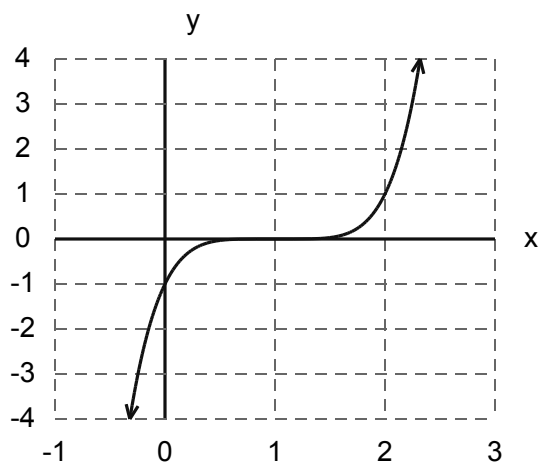
**At -1 it touches; at 1 it crosses with flattening.
Degree 5, negative leading coefficient: left up, right down.**

**PRACTICE 3**

**Cross at -2 and 0 (simple); touch at 2 (double).
Degree 4, both ends up. Negative only for $-2 < x < 0$.**

**PRACTICE 4**

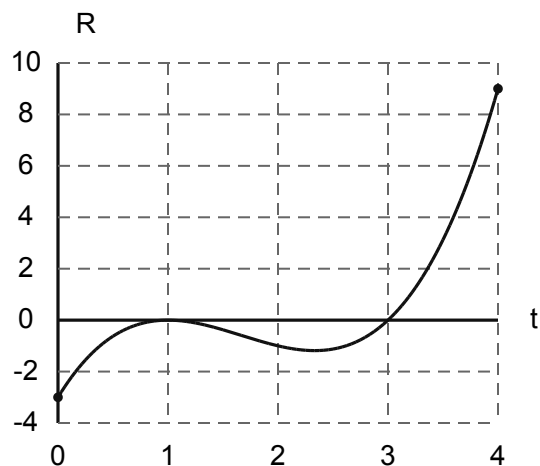
**Zero 1 has multiplicity 5: a flattened crossing.
Degree 5; left down and right up.**



APPLY IT 1

Touch at $t=1$; cross at $t=3$. Closed endpoints $(0,-3),(4,9)$.

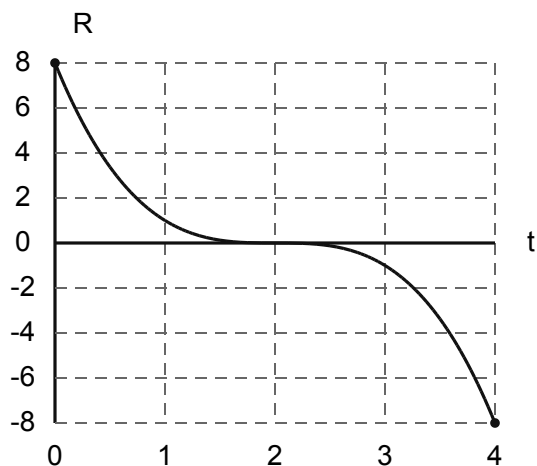
The repeated factor at 1 preserves the sign; the simple factor at 3 changes it.



APPLY IT 2

Flattened crossing at $t=2$; response decreases through zero. Closed endpoints $(0,8),(4,-8)$.

Multiplicity 3 is odd, so the sign changes despite the flat tangent.



CHALLENGE

Cross -2, touch 0, flattened cross 2. Degree 6, both ends down. Positive on $(-2,0)$ and $(0,2)$; exclude every zero.

