

Polynomial Pathways

Zeros, graphs, and real-world limits

HIGH SCHOOL • MA.912.AR.6.6

MA.912.AR.6.6 "Solve and graph mathematical and real-world problems that are modeled with polynomial functions of degree 3 or higher. Interpret key features and determine constraints in terms of the context."

NAME _____

DATE _____

POLYNOMIAL MODELS AND CONSTRAINTS

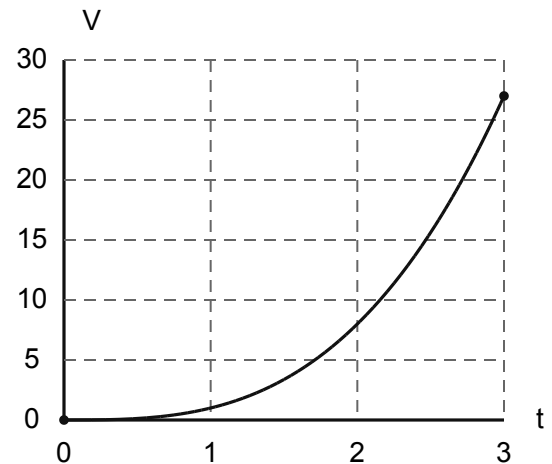
A polynomial model connects a graph, an equation and a situation. State the allowed input interval before solving. Use the graph to interpret zeros, maxima/minima, sign and increasing/decreasing intervals. Here all inputs vary continuously on the stated closed intervals. Use closed endpoint dots; do not extend a contextual model beyond its interval. Estimate noninteger turning points from the graph.

WORKED EXAMPLE

A tank model is $V(t)=t^3$ liters for $0 \leq t \leq 3$ minutes. Graph it and find when $V=8$.

1. Plot $(0,0),(1,1),(2,8),(3,27)$. Use closed endpoints.
2. Solve $t^3=8$ to obtain $t=2$ minutes.
3. The volume increases from 0 to 27 liters over the modeled time.

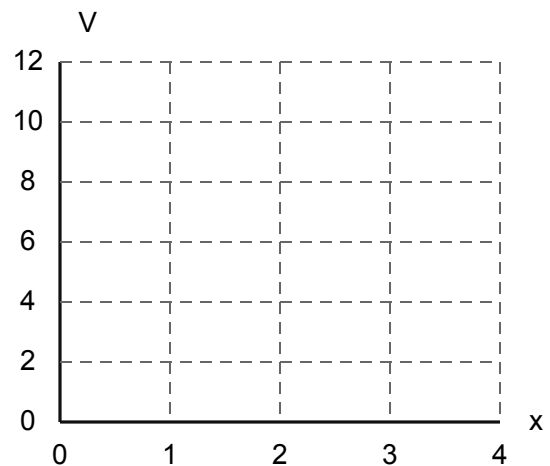
Answer: $t=2$ min; domain $[0,3]$; range $[0,27]$; increasing.



PRACTICE 4 problems

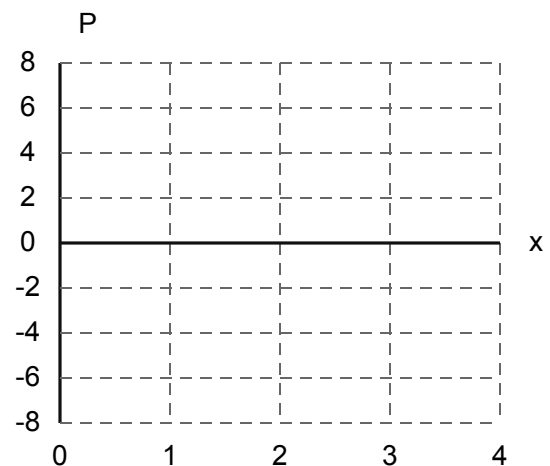
- 1** A simulated volume is $V(x)=x^2(4-x)$ for $0 \leq x \leq 4$; zero-volume end cases are included. Graph it. Find its zeros and estimate its maximum, domain and range.

ANSWER _____



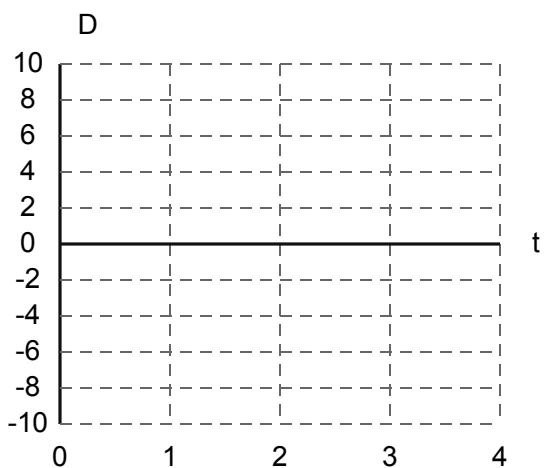
- 2** A continuous production model gives profit $P(x)=-(x-1)(x-2)(x-3)$, for $0 \leq x \leq 4$. Graph it. Give break-even inputs and all inputs with positive profit.

ANSWER _____



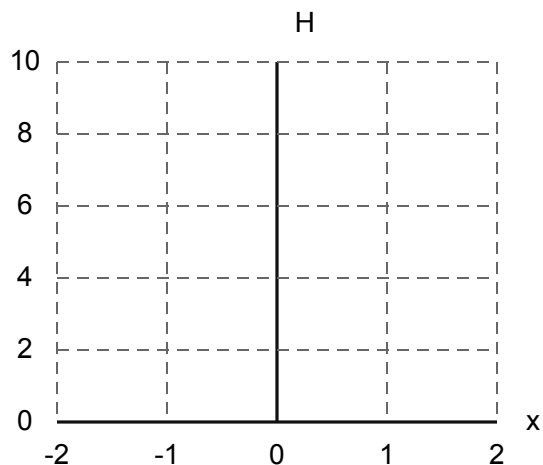
- 3** A calibration deviation is $D(t)=(t-2)^3-1$ for $0 \leq t \leq 4$ seconds. Graph it. Find its zero, range and monotonic behavior. Is its flat point a relative extremum?

ANSWER _____



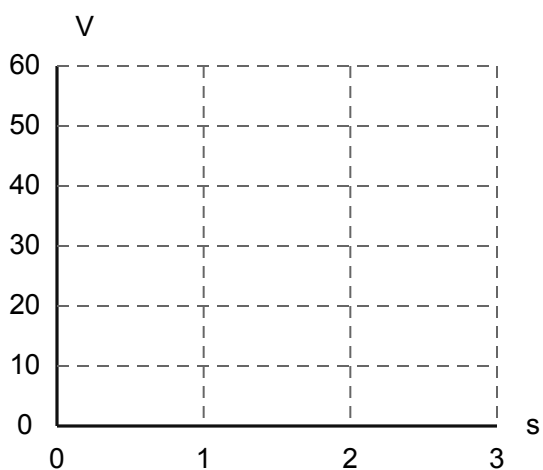
- 4** An arch profile is $H(x)=9-(x^2-1)^2$, for $-2 \leq x \leq 2$ meters. Graph it. Find ground points, maximum height, symmetry and the interior relative minimum.

ANSWER _____



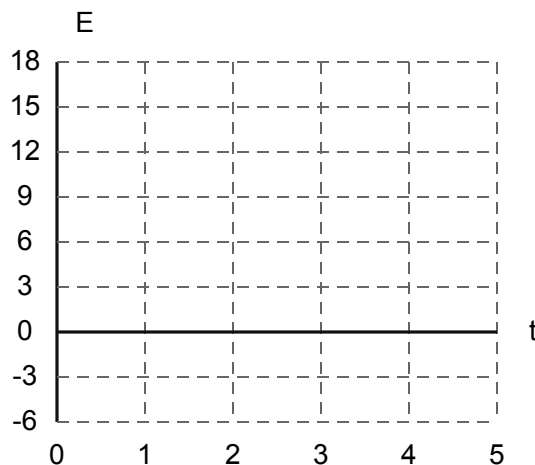
APPLY IT Show your work

- 1** A box has dimensions s , s and $2s$ centimeters, with $1 \leq s \leq 3$. Write and graph its volume $V(s)$. Find s and all dimensions when $V=16$. Give domain and range.



A sensor error is $E(t)=(t-1)^2(t-4)$, for $0 \leq t \leq 5$ seconds.

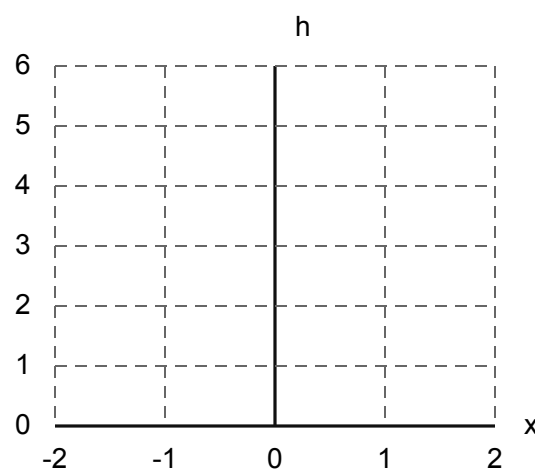
- 2** Graph it. Find zero times, intervals of positive error, and the range.



CHALLENGE One step up

A track profile is $h(x)=x^2(4-x^2)$, $-2 \leq x \leq 2$ meters. Graph it. Find zeros, symmetry, range, relative extrema and increasing/decreasing intervals; estimate irrational locations.

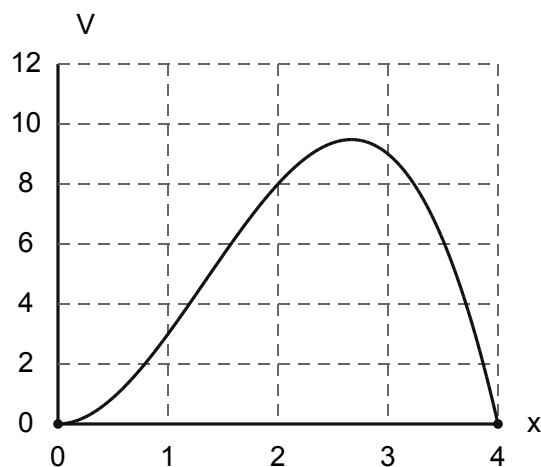
WORK SPACE AND ANSWER



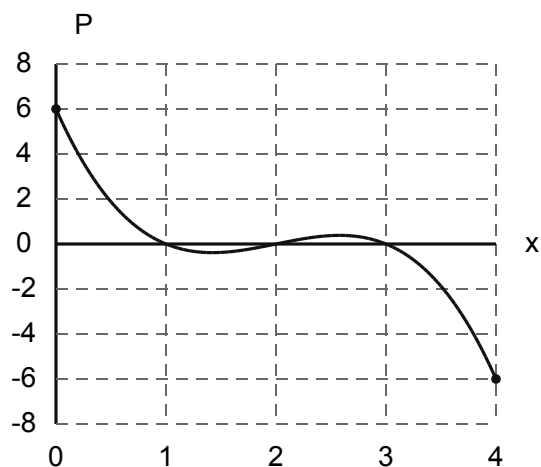
Require graphs, contextual constraints and units where supplied. Relative extrema here refer to interior turning points; compare endpoints separately for absolute extrema. Accept sensible graph estimates (about 2.67 and 1.41). No calculus is required. Include endpoints when a sign condition holds there.

PRACTICE 1

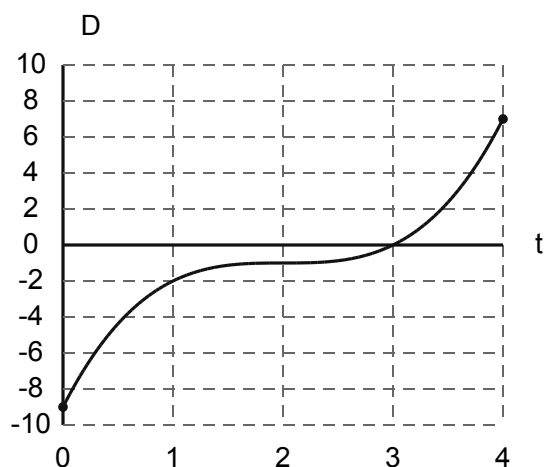
**Zeros 0,4; maximum about 9.48 at $x=2.67$.
Domain $[0,4]$, range approximately $[0,9.48]$.**

**PRACTICE 2**

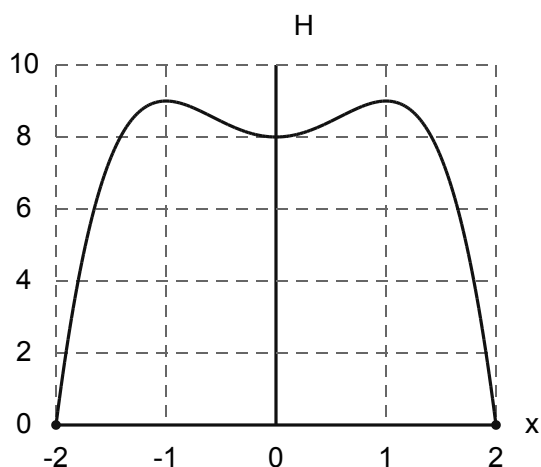
Break-even $x=1,2,3$. Positive on $[0,1)$ and $(2,3)$. Closed endpoints $(0,6)$, $(4,-6)$.

**PRACTICE 3**

**Zero $t=3$; range $[-9,7]$; increasing throughout.
The flat point $(2,-1)$ is not a relative extremum.**

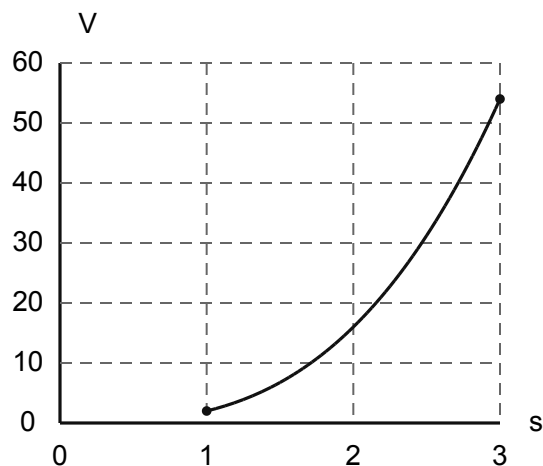
**PRACTICE 4**

Ground at $x=-2,2$; maximum 9 at $x=-1,1$. Y-axis symmetry; interior relative minimum $(0,8)$.



APPLY IT 1

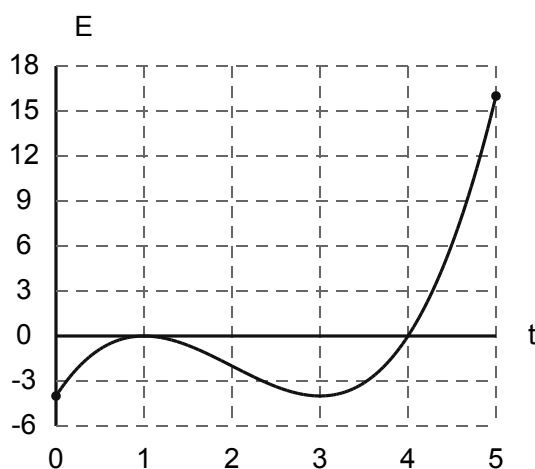
$V=2s^3$; $s=2$ cm; dimensions 2 by 2 by 4 cm.
Domain $[1,3]$; range $[2,54]$ cubic centimeters.
 $2s^3=16$ gives $s^3=8$ and $s=2$, inside the allowed interval.



APPLY IT 2

Zeros 1 (touch), 4 (cross); positive on $(4,5]$. Range $[-4,16]$; minimum -4 at $t=0$ and 3; maximum 16 at $t=5$.

Evaluate endpoints and turning points; the zero at 1 does not change sign.



CHALLENGE

Zeros -2,0,2; y-axis symmetry; range $[0,4]$. Maxima at $x \approx -1.41, 1.41$; local min $(0,0)$. Increasing $(-2,-1.41), (0,1.41)$; decreasing $(-1.41,0), (1.41,2)$.

