

Radical Function Paths

Graph, solve, and interpret square root or cube root models.

HIGH SCHOOL • MA.912.AR.7.3

MA.912.AR.7.3 "Solve and graph mathematical and real-world problems that are modeled with square root or cube root functions. Interpret key features and determine constraints in terms of the context."

NAME _____

DATE _____

RADICAL MODELS IN CONTEXT

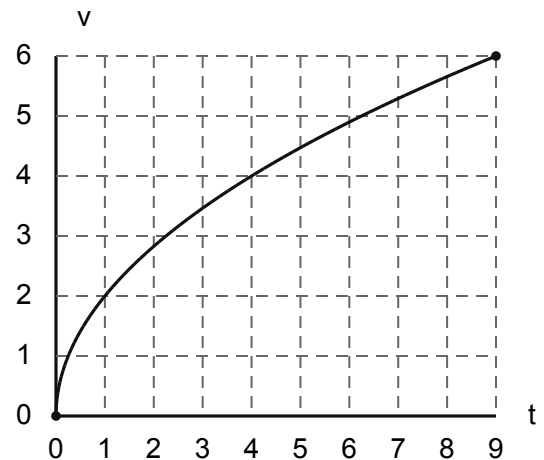
Translate the situation into a square-root or cube-root function, graph it on the allowed interval, and interpret solutions with units. Context can restrict a function more than its algebraic domain. Here all model inputs vary continuously. Closed endpoints are included. Check solutions after squaring: squaring can introduce values that fail the original equation.

WORKED EXAMPLE

A speed model is $v(t) = 2\sqrt{t}$ meters/second for $0 \leq t \leq 9$ seconds. Graph it and find when $v=4$.

1. Plot $(0,0)$, $(1,2)$, $(4,4)$, $(9,6)$ and connect smoothly on the allowed interval.
2. $2\sqrt{t}=4$ gives $\sqrt{t}=2$, so $t=4$ seconds.
3. The model increases from 0 to 6 meters/second.

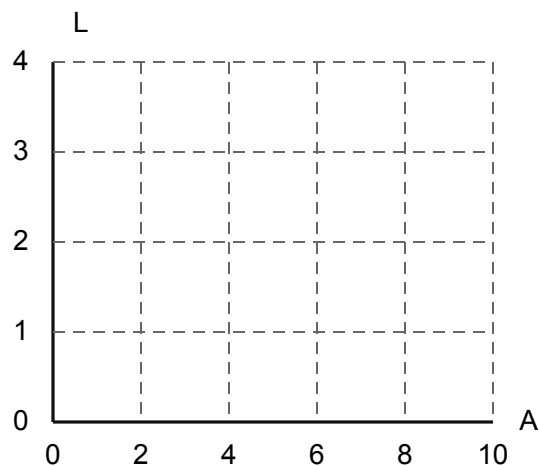
Answer: $t=4$ s; domain $[0,9]$; range $[0,6]$; closed endpoints.



PRACTICE 4 problems

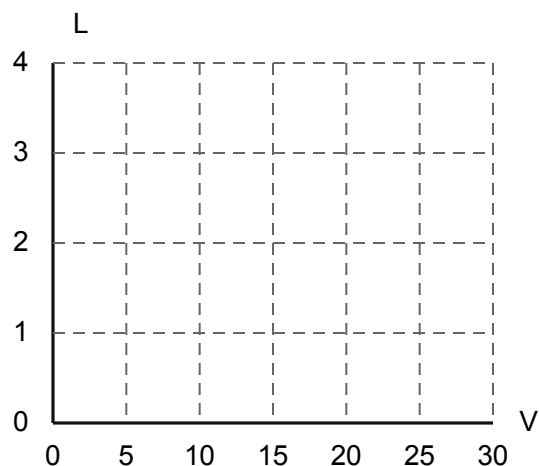
- 1** A square panel has area A square meters, $1 \leq A \leq 9$. Write and graph its side length $L(A)$. Find A when $L=2$ and give the range of L .

ANSWER _____



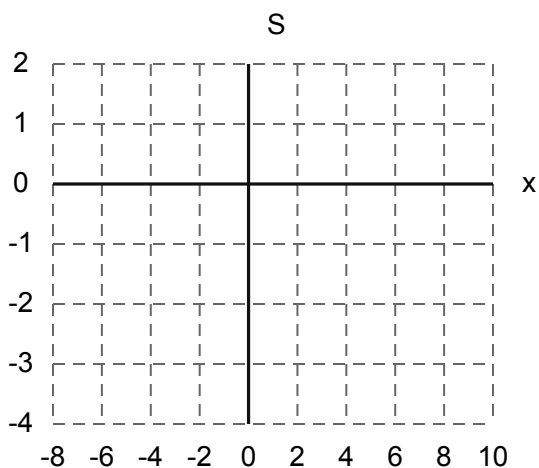
- 2** A cube has volume V cubic centimeters, $1 \leq V \leq 27$. Write and graph its edge length $L(V)$. Find V when $L=2$ and give the range of L .

ANSWER _____



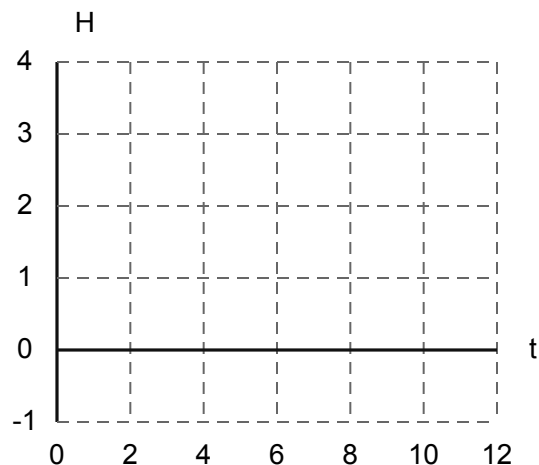
- 3** A real-valued control setting x gives reading $S = \sqrt[3]{(x-1)} - 1$ for $-7 \leq x \leq 9$. Graph it. Find the setting for $S=0$ and give the reading range.

ANSWER _____



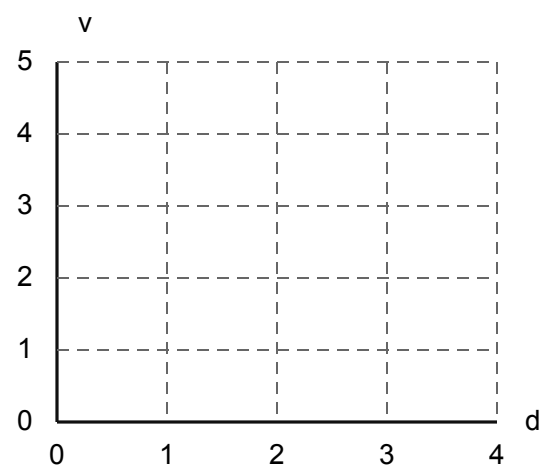
- 4** A meter reads $H(t)=3-\sqrt{t-2}$ for $2 \leq t \leq 11$ seconds. Graph it. When does $H=1$? Give its maximum, minimum and range.

ANSWER _____



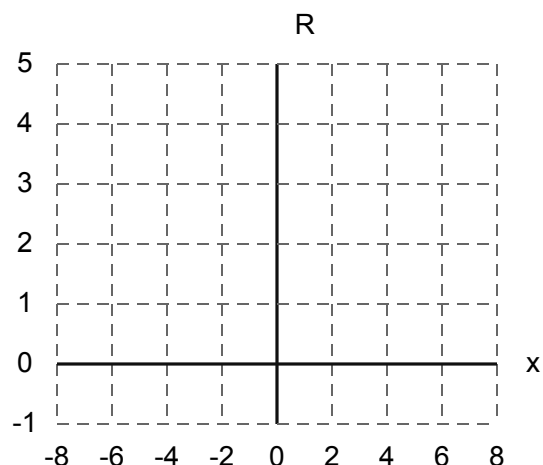
APPLY IT Show your work

- 1** A laboratory uses the model $v=\sqrt{4d}=2\sqrt{d}$, with depth $0 \leq d \leq 4$ meters and speed v in meters/second. Graph it. Find the depth for $v=3$ and give the speed range.



2

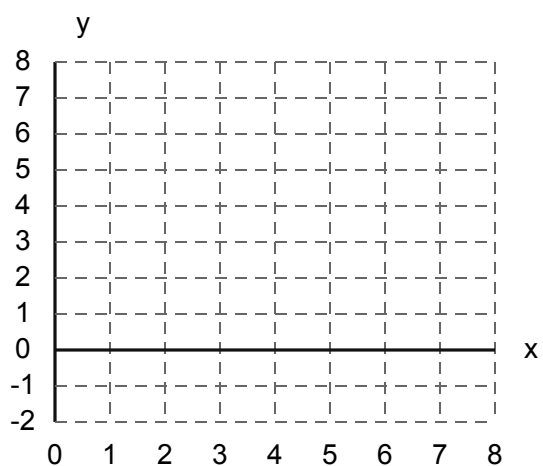
A display reads $R(x) = -\sqrt[3]{x} + 2$ for real control settings $-8 \leq x \leq 8$. Graph it. Find the setting for $R=3$ and describe its range and monotonic behavior.



CHALLENGE One step up

Two model readings are $y = \sqrt{x+1}$ and $y = x-1$ for $0 \leq x \leq 8$. Graph both on the same grid. Solve their equality algebraically and use the graph to reject any extraneous solution.

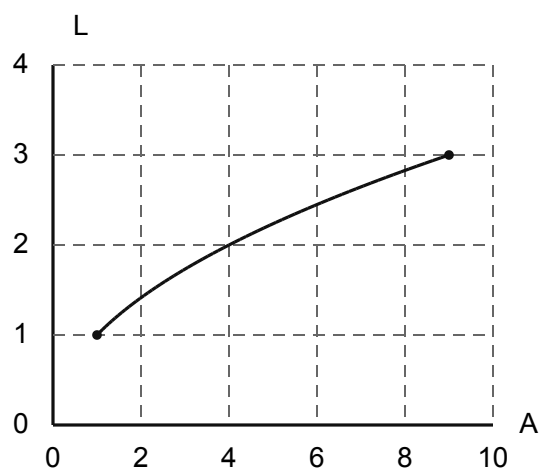
WORK SPACE AND ANSWER



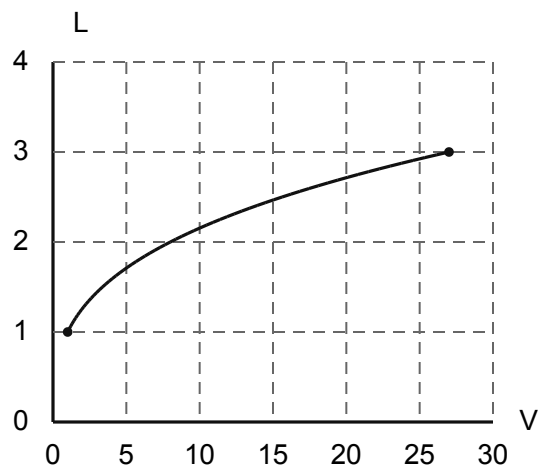
Require graphs on the stated continuous intervals, with closed endpoints and sensible units. All square-root values are nonnegative before shifts/reflections. Check radical equations in their original form. The challenge key uses a solid radical curve and dashed line.

PRACTICE 1

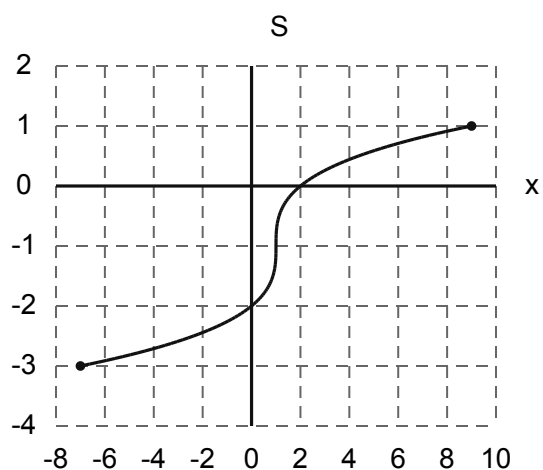
$L = \sqrt{A}$; $A = 4$ square meters; range $[1, 3]$ meters.

**PRACTICE 2**

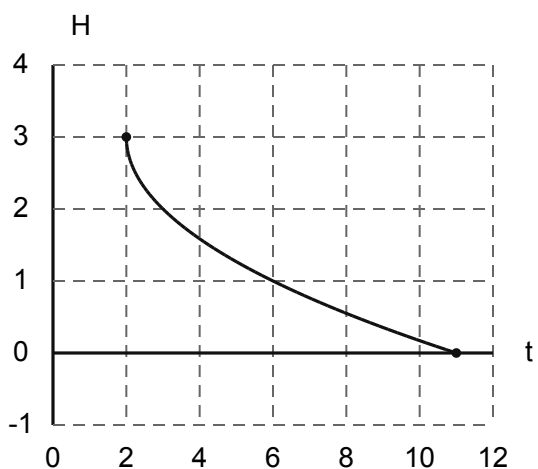
$L = \sqrt[3]{V}$; $V = 8$ cubic centimeters; range $[1, 3]$ centimeters.

**PRACTICE 3**

$S = 0$ at $x = 2$; range $[-3, 1]$; increasing.

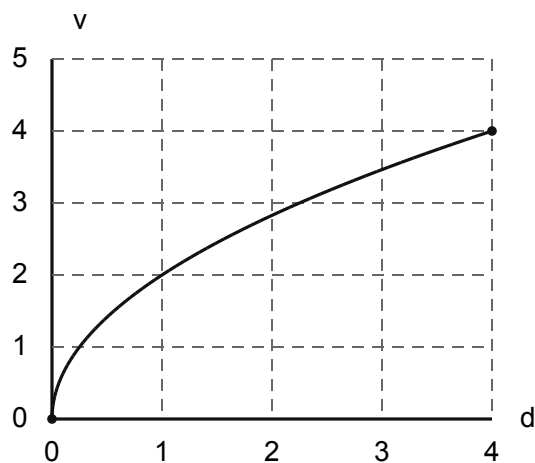
**PRACTICE 4**

$H = 1$ at $t = 6$; maximum 3 at $t = 2$; minimum 0 at $t = 11$; range $[0, 3]$.



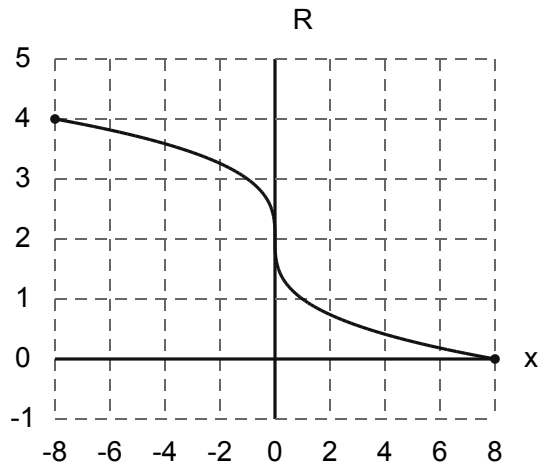
APPLY IT 1

Depth $d=2.25$ m; speed range $[0,4]$ m/s.
 $2\sqrt{d}=3$ gives $d=(3/2)^2=2.25$, inside $[0,4]$.



APPLY IT 2

$x=-1$; range $[0,4]$; decreasing throughout.
 $-\sqrt[3]{x}+2=3$ gives $\sqrt[3]{x}=-1$, hence $x=-1$.



CHALLENGE

**Only $x=3$, intersection $(3,2)$. Squaring gives $x=0$ or 3 ;
 $x=0$ fails because 1 is not -1 .**

