

Rational Graph Detective

Solve equations, read functions, and spot restrictions.

HIGH SCHOOL • MA.912.AR.8

MA.912.AR.8 "Solve and graph rational equations and functions in one and two variables."

NAME _____

DATE _____

RATIONAL FUNCTIONS: EQUATIONS AND GRAPHS

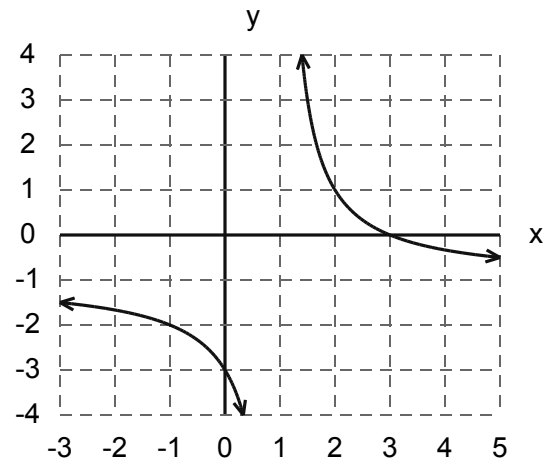
A rational function is a quotient of polynomials. Exclude inputs making its original denominator zero, even after cancellation. Graph disconnected branches separately. An asymptote describes limiting behavior; a canceled factor can leave a hole. Solve equations, then check restrictions. Inverse variation has form $y=k/x$. Context determines whether inputs are continuous or discrete.

WORKED EXAMPLE

Graph $y=2/(x-1)-1$ and solve $y=1$. Give the domain, range and asymptotes.

1. Exclude $x=1$. The vertical asymptote is $x=1$ and horizontal asymptote is $y=-1$.
2. Plot points on each side of $x=1$; never connect across it.
3. $2/(x-1)-1=1$ gives $x=2$; substitution confirms $y=1$.

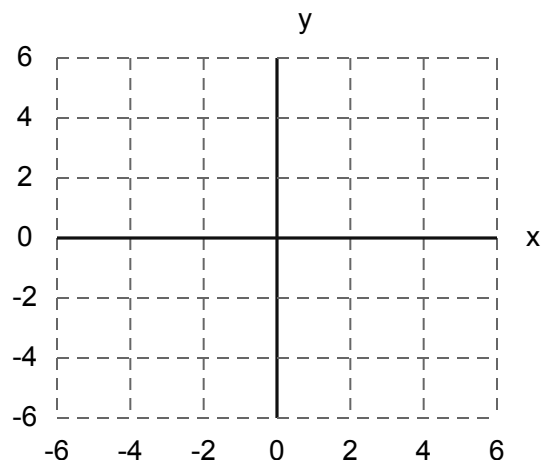
Answer: $x=2$; domain $x \neq 1$; range $y \neq -1$; asymptotes $x=1, y=-1$.



PRACTICE 4 problems

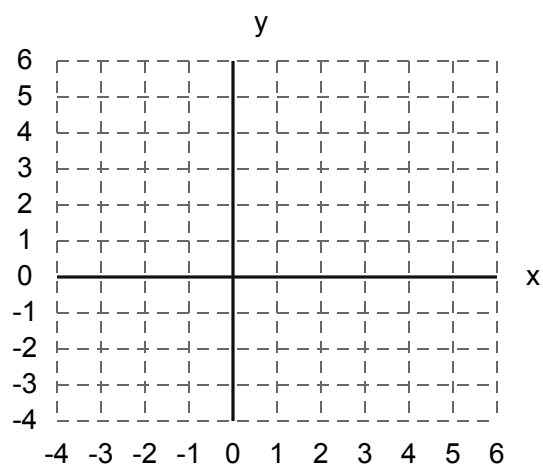
- 1** Graph $y=4/x$ and solve $4/x=2$. Give domain/range and both asymptotes.

ANSWER _____



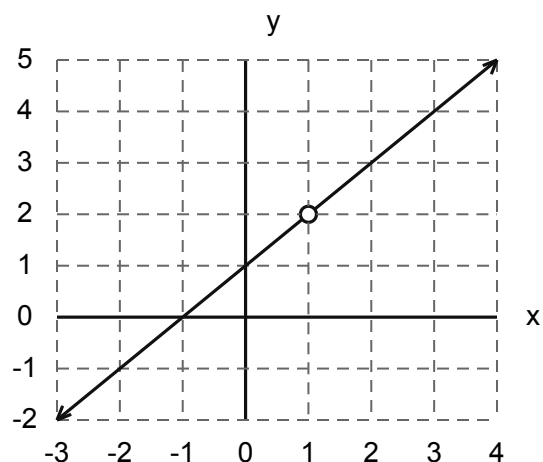
- 2** Graph $y=(x+2)/(x-1)=1+3/(x-1)$. Solve $y=3$ and give domain/range and asymptotes.

ANSWER _____



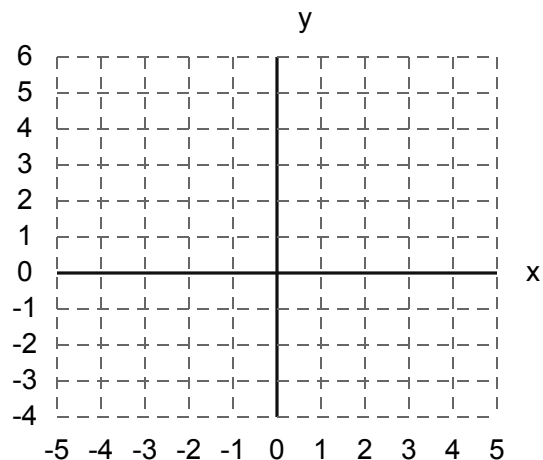
- 3** The graph represents $y=(x^2-1)/(x-1)$. Explain the hole using the original denominator. Solve $y=2$, and give domain and range.

ANSWER _____



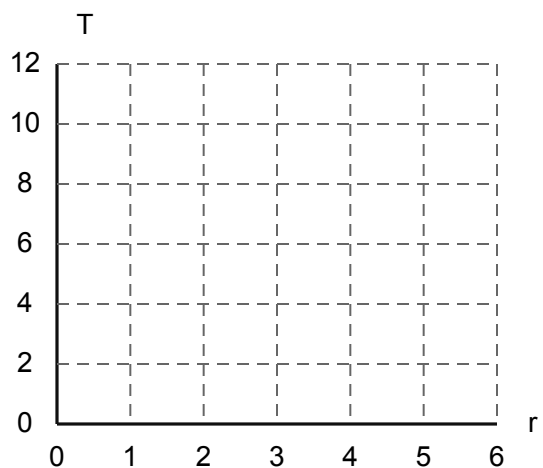
- 4** A curve $y = a/(x+1) + 2$ contains $(-3, 3), (-2, 4), (0, 0), (1, 1)$. Find a and graph it. Give its asymptotes and x -intercept.

ANSWER _____



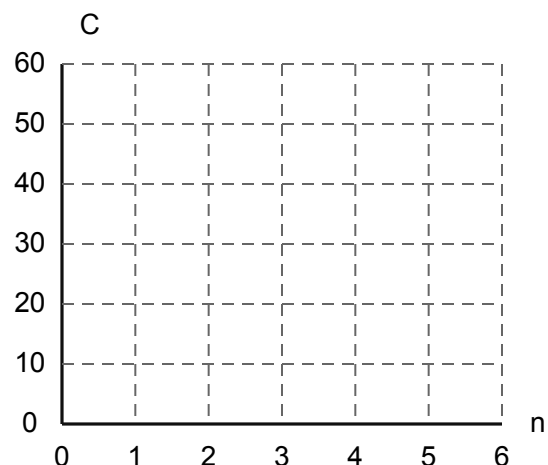
APPLY IT Show your work

- 1** A pump moves 12 liters at a continuous rate r liters/minute, $1 \leq r \leq 6$. Write and graph the time $T(r)$. Find r when $T=3$ minutes, and give the time range.



2

A \$60 rental is split equally among n people, where n is an integer from 1 to 6. Plot the possible costs $C=60/n$. How many people give a \$15 cost each? State domain and range.

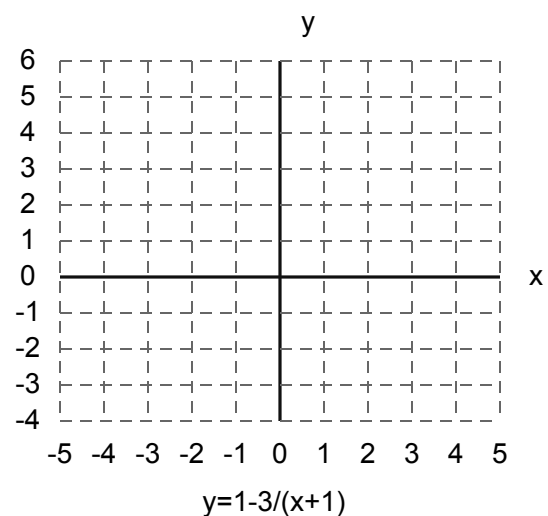


CHALLENGE

One step up

Graph $y=(x-2)/(x+1)=1-3/(x+1)$. Use the graph and sign reasoning to solve $(x-2)/(x+1) \geq 0$. Identify excluded and included boundary values.

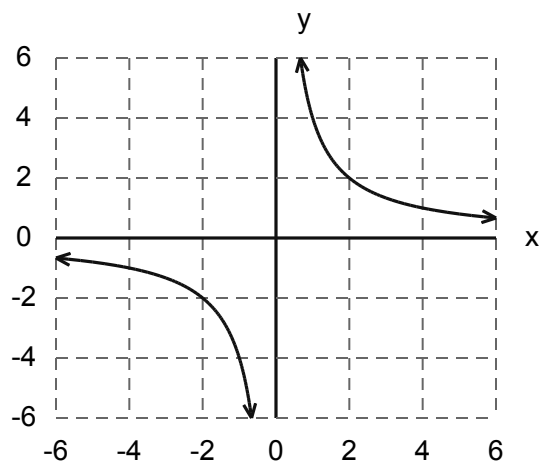
WORK SPACE AND ANSWER



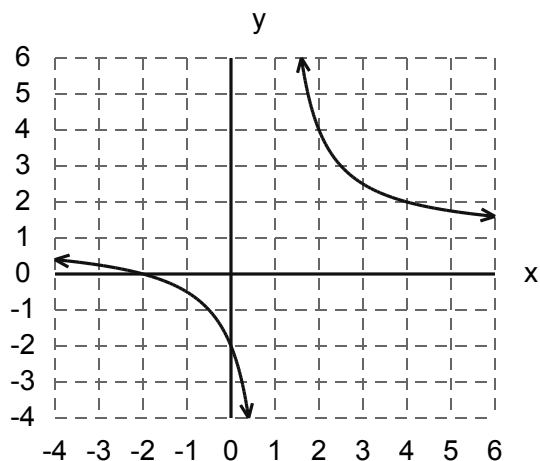
Require labeled asymptotes, separate branches and continuation arrows for unrestricted curves. Draw a hollow circle at the canceled-factor hole. A grid line at an asymptote is not part of the curve. The pump uses a closed continuous interval; the rental uses only six discrete points.

PRACTICE 1

$x=2$; domain $x \neq 0$; range $y \neq 0$; asymptotes $x=0, y=0$.

**PRACTICE 2**

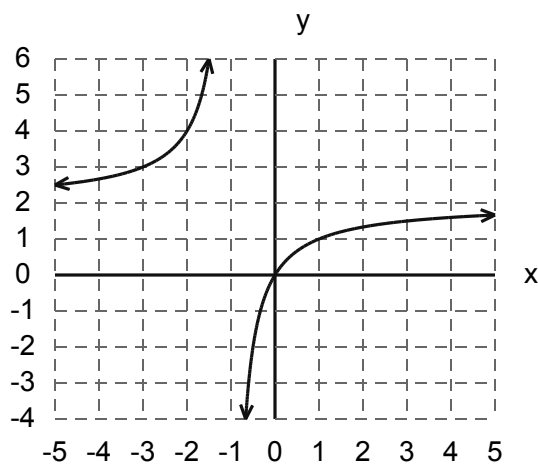
$x=2.5$; domain $x \neq 1$; range $y \neq 1$; asymptotes $x=1, y=1$.

**PRACTICE 3**

The line $y=x+1$ has a hole at $(1,2)$. No solution to $y=2$: $x=1$ is excluded. Domain $x \neq 1$; range $y \neq 2$.

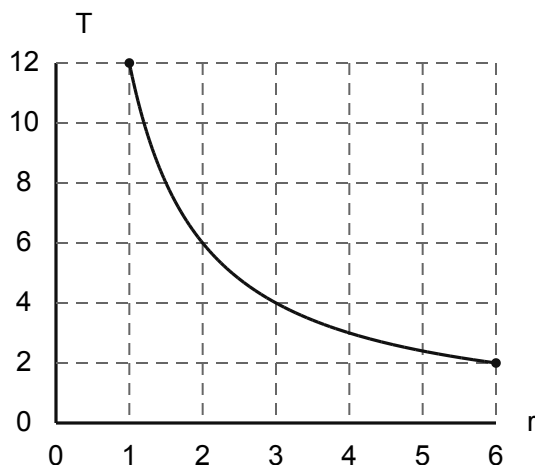
PRACTICE 4

a=-2; asymptotes $x=-1, y=2$; x-intercept (0,0).



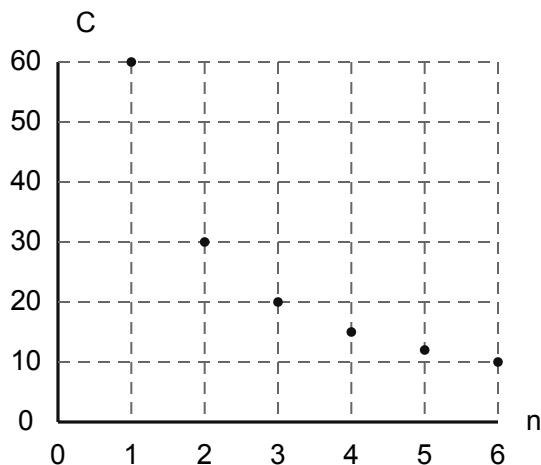
APPLY IT 1

$T = 12/r$; $r = 4$ L/min; time range $[2, 12]$ minutes.
The product $rT = 12$ is fixed. $12/r = 3$ gives $r = 4$, inside $[1, 6]$.



APPLY IT 2

$n=4$. Domain $\{1, 2, 3, 4, 5, 6\}$; range $\{\$60, \$30, \$20, \$15, \$12, \$10\}$. Plot six separate points, with no connecting curve.
 $60/n = 15$ gives $n=4$, an allowed whole-number count.



CHALLENGE

Solution $x < -1$ or $x \geq 2$. Exclude -1 (denominator zero); include 2 (function zero). Asymptotes $x = -1, y = 1$.

