

Closing In

Estimate limits from graphs and nearby table values.

HIGH SCHOOL · MA.912.C.1.1

MA.912.C.1.1 "Demonstrate understanding of the concept of a limit and estimate limits from graphs and tables of values."

NAME _____

DATE _____

WHAT VALUE DOES THE GRAPH APPROACH?

A limit describes the output approached as the input gets close to a target from both sides. Compare left-hand and right-hand behavior: a two-sided limit exists only when they agree. A solid dot gives the value at the target; a hollow circle excludes that point. The point value can differ from the limit or be undefined. Tables support estimates, not proof of every possible nearby value.

WORKED EXAMPLE

The table shows values near $x=2$; $f(2)$ is undefined. Estimate the limit as x approaches 2.

VALUES NEAR $x=2$

x	$f(x)$
1.9	4.9
1.99	4.99
2.01	5.01
2.1	5.1

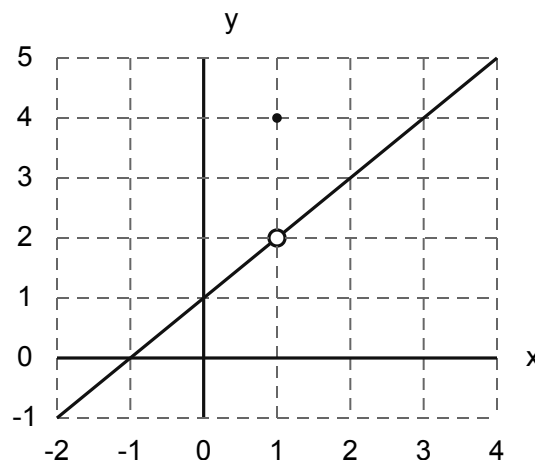
1. From the left, the outputs approach 5 from below.
2. From the right, the outputs approach 5 from above.
3. Both sides approach the same value; the missing point does not prevent a limit.

Answer: Estimated limit 5, although $f(2)$ is undefined.

PRACTICE 4 problems

- 1** Use the graph near $x=1$. Estimate the left-hand limit, right-hand limit and two-sided limit. What is $f(1)$? Explain the difference.

ANSWER _____



- 2** Use the table to estimate the limit as x approaches -1 . Can this table alone tell you $f(-1)$? Explain.

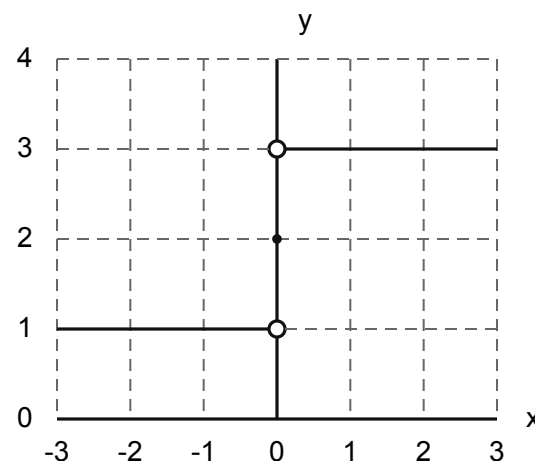
ANSWER _____

VALUES NEAR $x=-1$

x	$f(x)$
-1.1	3.21
-1.01	3.0201
-0.99	2.9801
-0.9	2.81

- 3** Use the graph near $x=0$. Estimate both one-sided limits and $f(0)$. Does the two-sided limit exist? Explain.

ANSWER _____



- 4 The table shows values near $x=4$, but $f(4)=-2$. Estimate the two-sided limit. Does the given point value change your estimate?

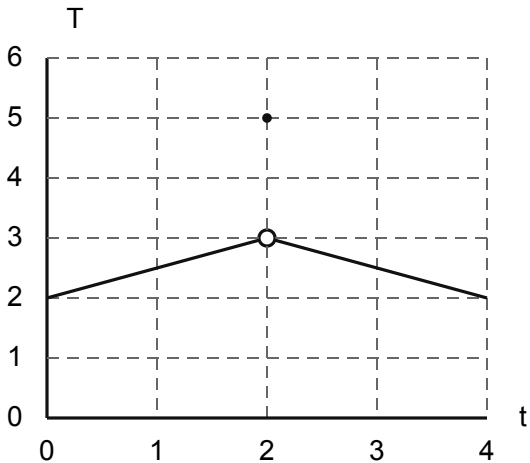
ANSWER _____

VALUES NEAR $x=4$

x	$f(x)$
3.9	7.8
3.99	7.98
4.01	8.02
4.1	8.2

APPLY IT Show your work

- 1 The graph shows a temperature reading T near $t=2$ seconds. What temperature does it approach from each side? What reading is recorded exactly at $t=2$? Explain.



- 2 A flow sensor misses its reading at $t=3$ seconds. Use the nearby readings in the table to estimate the flow-rate limit in liters/minute. Does the missing reading prevent that estimate?

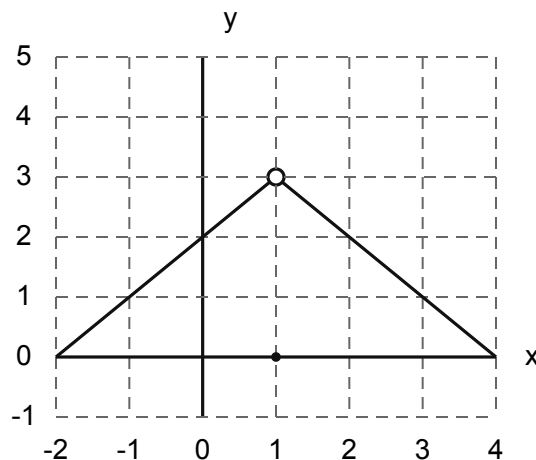
FLOW NEAR $T=3$ SECONDS

t (s)	Rate (L/min)
2.9	4.9
2.99	4.99
3.01	5.01
3.1	5.1

CHALLENGE One step up

Use the graph near $x=1$. Give the limit and $f(1)$. Would moving only the solid dot to $(1,3)$ change the limit? Starting from the original graph, what if only the right branch moved up 1 unit?

WORK SPACE AND ANSWER



Judge estimates using the supplied graphs and tables. Require separate left/right reasoning for a jump. DNE means the two-sided limit does not exist. Do not confuse a plotted point with nearby behavior. Graphs show the local neighborhoods relevant to each question; no global domain is asserted.

PRACTICE 1

Both one-sided limits and the two-sided limit are 2; $f(1)=4$. Nearby values approach the hole, not the isolated solid dot.

PRACTICE 2

Estimated limit 3 from both sides. No: the table lists nearby inputs but never $x=-1$.

PRACTICE 3

Left limit 1; right limit 3; $f(0)=2$. The two-sided limit does not exist because the one-sided limits differ.

PRACTICE 4

Estimated limit 8. The isolated value $f(4)=-2$ does not change the nearby trend.

APPLY IT 1

Both sides approach 3 degrees; the recorded value at $t=2$ is 5 degrees. Changing only that single reading does not change the limit.

Trace each branch toward $t=2$, then read the separate solid dot.

APPLY IT 2

Estimated limit 5 L/min. The nearby readings approach 5 from both sides despite the missing reading. Values just below 3 approach 5 from below; values just above 3 approach 5 from above.

CHALLENGE

Original limit 3; $f(1)=0$. Moving only the dot keeps limit 3. Raising only the right branch gives left limit 3 and right limit 4, so no two-sided limit.