

Rational Graph Detective

Use zeros, asymptotes and points to build each graph.

HIGH SCHOOL • HSF-IF.C.7d

CCSS.Math.Content.HSF-IF.C.7d "(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior."

NAME _____

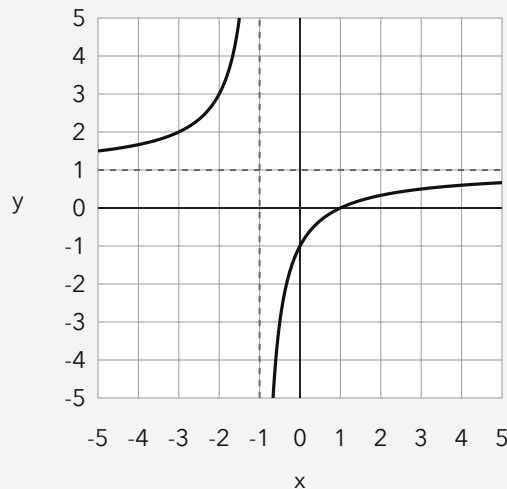
DATE _____

FROM FACTORS TO A GRAPH

Factor first and keep every original denominator restriction. Fully cancel common factors. A restricted input is a hole if the reduced expression is finite there; a remaining denominator zero gives a vertical asymptote. Find numerator zeros within the original domain. Compare degrees for horizontal end behavior. Plot points on each side of each vertical asymptote, then draw separate branches approaching dashed guides. A guide is not part of the graph.

WORKED EXAMPLE

Graph $y = (x - 1)/(x + 1)$. Identify its zero, asymptotes and end behavior.

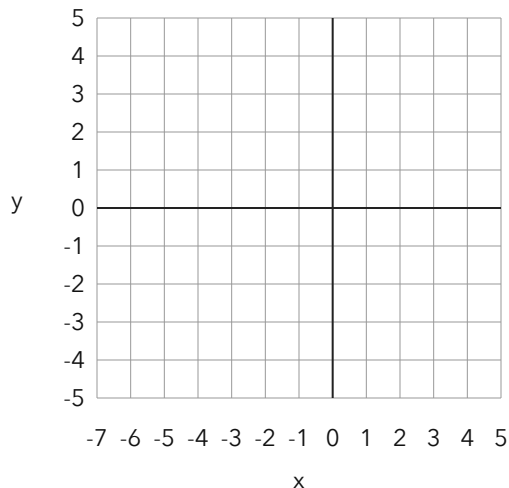


1. The zero is $(1, 0)$, and the vertical asymptote is $x = -1$.
2. Equal leading coefficients give horizontal asymptote $y = 1$. The y -intercept is $(0, -1)$.
3. Rewrite $y = 1 - 2/(x + 1)$. Points $(-2, 3)$ and $(1, 0)$ locate separate branches.
4. As x tends to negative infinity, y approaches 1 from above; as x tends to positive infinity, it approaches 1 from below.

Answer: Zero $(1, 0)$; vertical guide $x = -1$; horizontal guide $y = 1$. Dashed guides are not function values.

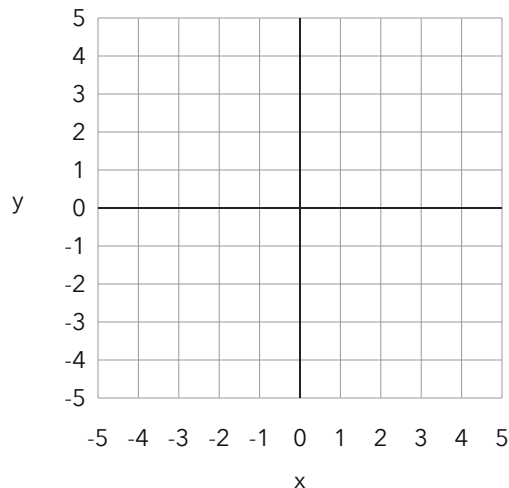
PRACTICE 4 problems

- 1** Graph $y = (x - 1)/(x + 3)$. State its zero, asymptotes and end behavior.



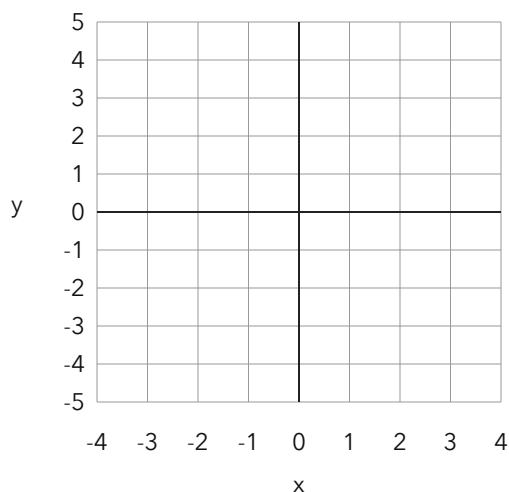
ANSWER _____

- 2** Graph $y = (x + 2)/(x - 1)$. State its zero, asymptotes and end behavior.



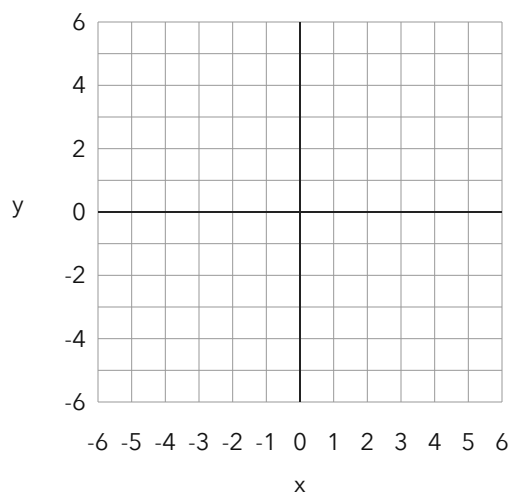
ANSWER _____

- 3** Graph $y = 2/(x^2 - 1)$. State zeros, asymptotes and end behavior.



ANSWER _____

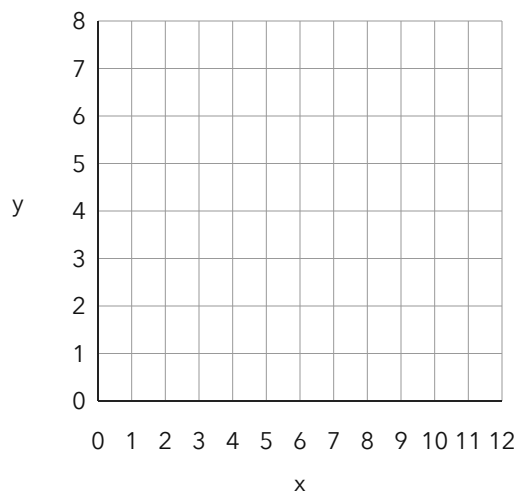
- 4** Graph $y = 2(x - 2)/(x + 2)$. State its zero, asymptotes and end behavior.



ANSWER _____

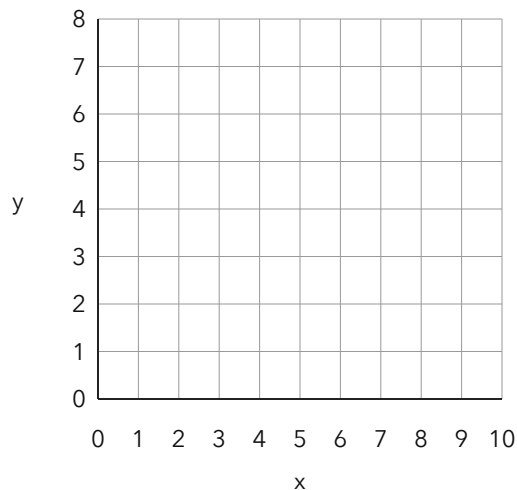
APPLY IT Show your work**1**

A print shop charges a \$6 setup fee plus \$1 per meter of banner. For a positive length x meters, average cost per meter is $C(x) = (x + 6)/x$. Graph the model for $x > 0$. Describe its asymptotes, end behavior and whether a zero is meaningful.

**REASONING AND ANSWER**

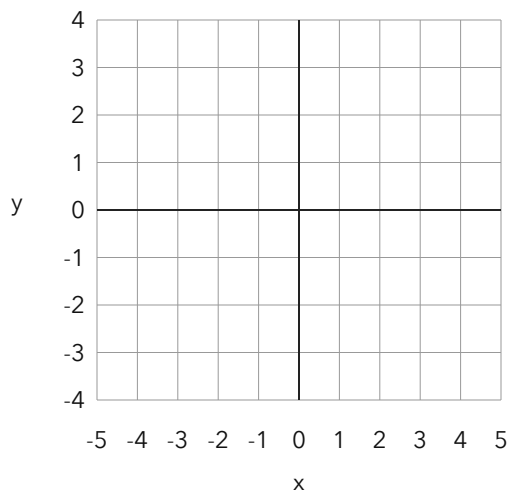
2

A lab models concentration after mixing by $C(x) = 6x/(x + 2)$, where $x > 0$ is added liquid in liters. Graph C for $x > 0$. Identify its limiting value and explain why the apparent zero at $x = 0$ is excluded.

**REASONING AND ANSWER**

CHALLENGE One step up

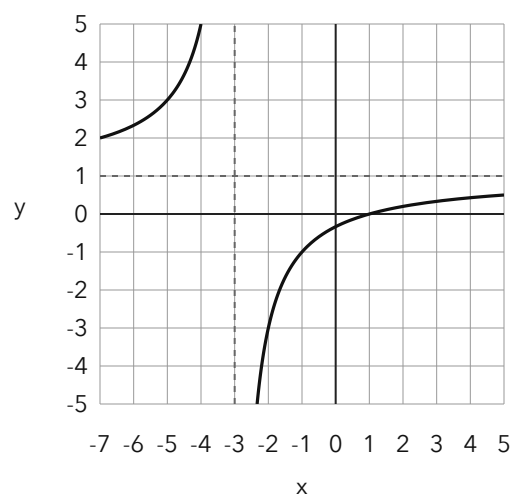
Graph $y = ((x - 2)(x + 1))/((x - 2)(x + 2))$. Mark the hole and identify the zero, asymptotes and end behavior. Explain the difference between a hole and a vertical asymptote.



WORK SPACE AND ANSWER

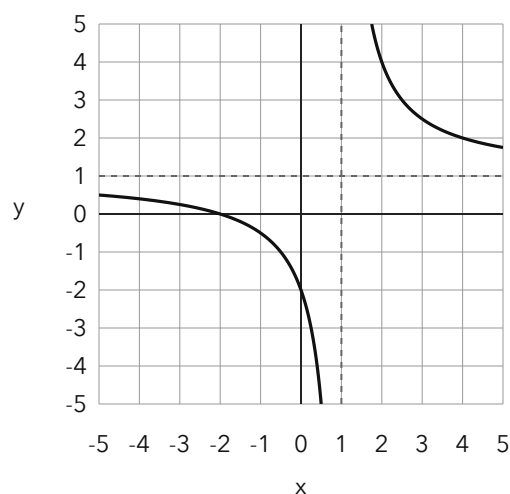
PRACTICE 1

Zero $(1, 0)$; vertical $x = -3$; horizontal $y = 1$. As x tends to negative infinity, y approaches 1 from above; toward positive infinity, from below. Points $(-4, 5)$, $(-2, -3)$ and $(0, -1/3)$.



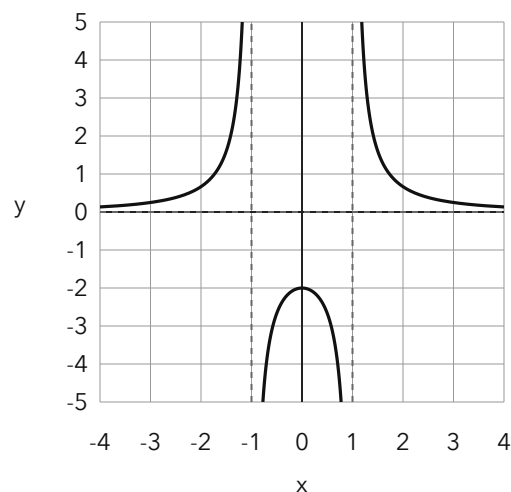
PRACTICE 2

Zero $(-2, 0)$; vertical $x = 1$; horizontal $y = 1$. At negative infinity approach 1 from below; at positive infinity from above. Points $(0, -2)$, $(2, 4)$.



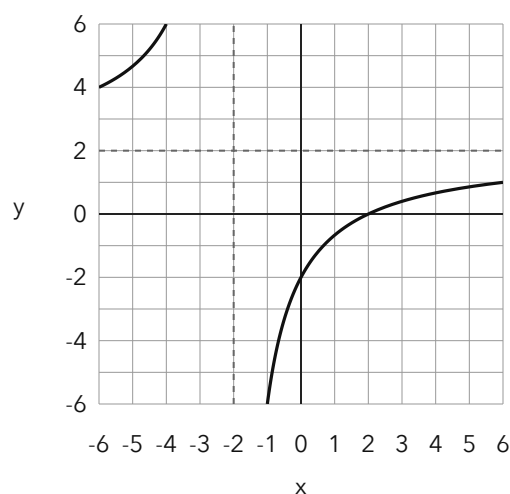
PRACTICE 3

No zeros. Verticals $x = -1$ and $x = 1$; horizontal $y = 0$. Both outer branches approach 0 from above at the ends. The middle branch is negative and passes through $(0, -2)$; outer points $(\pm 2, 2/3)$.



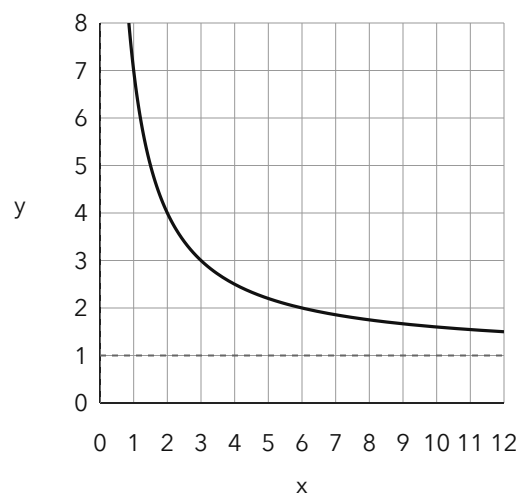
PRACTICE 4

Zero $(2, 0)$; vertical $x = -2$; horizontal $y = 2$. At negative infinity approach 2 from above; at positive infinity from below. Points $(0, -2)$, $(-4, 6)$.



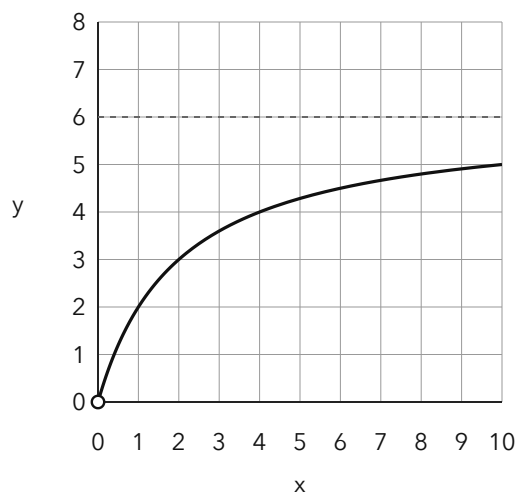
APPLY IT 1

No meaningful zero. Vertical $x = 0$; horizontal $C = 1$. As x grows, average cost approaches \$1 per meter from above. $C = 1 + 6/x$; points (1, 7), (2, 4), (6, 2). At x near 0 from the right, C grows without bound.



APPLY IT 2

C approaches 6 from below as x grows. There is no zero in the stated $x > 0$ domain. The boundary point (0, 0) is open; no vertical asymptote lies in or at this domain. $C = 6 - 12/(x + 2)$; points (1, 2), (2, 3), (4, 4). The algebraic vertical asymptote $x = -2$ is outside the applied domain.



CHALLENGE

Simplify to $(x + 1)/(x + 2)$, but $x \neq 2$ and -2 . Hole (2, 3/4); zero (-1, 0); vertical $x = -2$; horizontal $y = 1$. At negative infinity approach 1 from above; at positive infinity from below. At $x = 2$ the limit is finite, whereas values grow without bound on either side of $x = -2$.

Student grids contain no curves or guides. Require sketches with distinct branches, dashed asymptotes, correct intercepts and approach directions. The challenge needs an open circle, not an asymptote, at the canceled input. Applied domains describe continuous cost and concentration models with $x > 0$; do not extend them into negative inputs.

