

Squares Tell Why

Explain the theorem and why its converse is true.

GRADE 8 • 8.GSR.8.1

8.GSR.8.1 "Explain a proof of the Pythagorean Theorem and its converse using visual models."

NAME _____

DATE _____

REASON WITH THE MODEL

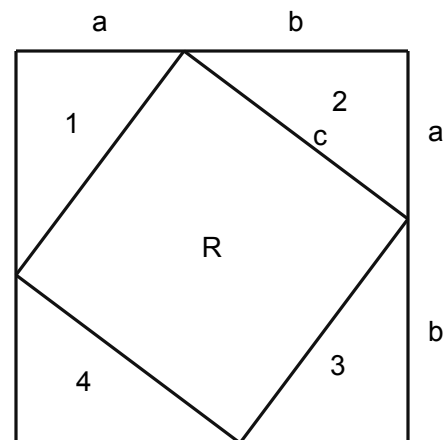
Let a and b be the legs of any right triangle and c its hypotenuse. Four congruent copies can be rearranged inside a square of side $a + b$. A proof explains why the result holds for every such triangle; checking a few side lengths is not a general proof. The drawings use one illustrative shape, with symbolic lengths.

WORKED EXAMPLE

In Arrangement A, identify the total area and the area occupied by triangles 1-4.

1. Each outer side is $a + b$, so its square has area $(a + b)^2$.
2. Each numbered triangle has area $ab/2$. All four occupy $2ab$.
3. Every side of the central region is a triangle hypotenuse, of length c .

Answer: Outer-square area: $(a + b)^2$. Total area of the four triangles: $2ab$.

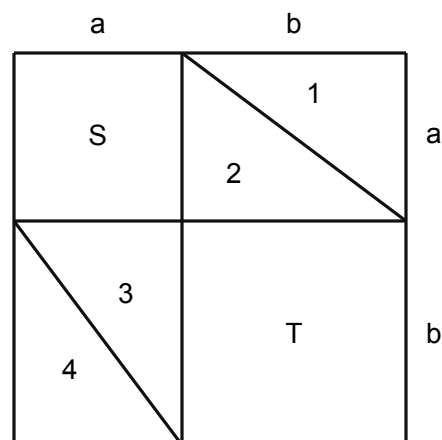


Arrangement A

PRACTICE 4 problems

- 1** Refer to Arrangement A above. Let the two acute angles of one small triangle be α and β . Explain why $\alpha + \beta = 90^\circ$, then why each angle of the central region R is 90° . Conclude the area of R .

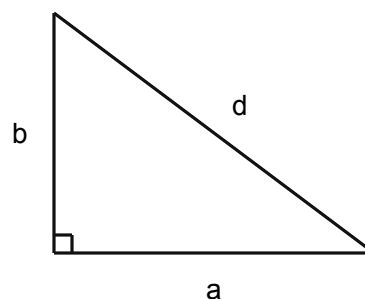
- 2** Arrangement B uses the same outer square and four numbered triangles. Find the areas of square regions S and T. Explain why their combined area is $a^2 + b^2$.



Arrangement B

- 3** Use Arrangements A and B. Write one equation for the equal leftover areas. Explain why rearranging triangles preserves that area, and why the argument works for any positive leg lengths.

- 4** A triangle has sides a , b , c with $a^2 + b^2 = c^2$. Consider the constructed right triangle shown, with legs a , b and hypotenuse d . Use the theorem to compare d and c . Then use SSS (equal corresponding sides give congruent triangles) to prove the original triangle is right.



Constructed right triangle

APPLY IT Show your work

- 1** A builder checks a triangular frame with sides 6 m, 8 m and 10 m. Explain why constructing a right triangle with legs 6 and 8 proves that the frame has a right angle; include the side comparison.

2

A student checks $3^2 + 4^2 = 5^2$ and says this proves the Pythagorean Theorem for every right triangle. Explain what the calculation establishes and what the rearrangement proof adds.

CHALLENGE

One step up

State the starting assumption and conclusion of the theorem, then of its converse. Identify the different visual model used for each proof on this sheet.

Accept equivalent area, angle and SSS arguments. Arrangements A and B each contain the same four right triangles, labeled 1-4, with legs a, b and hypotenuse c ; labels identify pieces, not correspondence of position. The converse proof must compare with a constructed right triangle, not assume the original is right. No cutting or measurement is required.

PRACTICE 1

The angles of a right triangle total 180° , so $\alpha + \beta = 90^\circ$. Along each straight outer edge, the central angle is $180^\circ - (\alpha + \beta) = 90^\circ$. Its four sides are c , so it is a square of area c^2 .

PRACTICE 2

Upper-left S has side a and area a^2 ; lower-right T has side b and area b^2 . They do not overlap. The four triangles cover the other two rectangles, so the leftover area is $a^2 + b^2$.

PRACTICE 3

Both leftovers equal $(a + b)^2 - 4(ab/2)$. Therefore $c^2 = a^2 + b^2$. The outer areas and the removed triangle areas are equal; moving congruent pieces changes no area. The argument uses arbitrary positive a and b , not special measurements.

PRACTICE 4

For the constructed right triangle, $d^2 = a^2 + b^2 = c^2$. Since lengths are positive, $d = c$. The original and constructed triangles have equal sides a, b, c , so SSS makes them congruent. The angle between a and b in the original is therefore 90° .

APPLY IT 1

The constructed right triangle has hypotenuse d with $d^2 = 6^2 + 8^2 = 100$, so $d = 10$. Its three sides match the frame. By SSS they are congruent, so the frame has a right angle between its 6 m and 8 m sides. This applies the converse proof rather than simply assuming that the frame is right.

APPLY IT 2

The calculation verifies one set of side lengths; it does not cover all right triangles. The rearrangement proof uses arbitrary positive legs a and b and equal areas to establish $c^2 = a^2 + b^2$ generally. A numerical example can illustrate a theorem, but cannot by itself establish the universal claim.

CHALLENGE

Theorem: start with a right triangle; conclude $a^2 + b^2 = c^2$ using the two square dissections.

Converse: start with $a^2 + b^2 = c^2$ for triangle sides; conclude the angle between a and b is right by constructing a right triangle and comparing all three sides.