

Points Tell the Story

Show how an equation becomes a complete set of solutions.

GRADE 8 • 8.PAR.4.2

8.PAR.4.2 "Show and explain that the graph of an equation representing an applicable situation in two variables is the set of all its solutions plotted in the coordinate plane."

NAME _____

DATE _____

REASON WITH THE MODEL

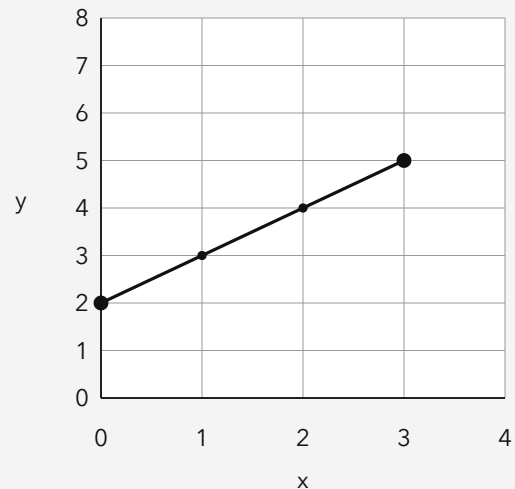
A point belongs to an equation's graph exactly when its coordinates satisfy the equation. A situation may restrict allowable inputs. Measured time can include fractions, so its graph can contain a whole segment; a count of whole objects gives separate points. A few sample points may not be the complete graph.

WORKED EXAMPLE

A tank starts with 2 liters and gains 1 liter per minute for 3 minutes. Its model is $y = x + 2$, $0 \leq x \leq 3$. Explain why its graph is a segment, not just the marked points.

1. The marked points $(0, 2)$, $(1, 3)$, $(2, 4)$, $(3, 5)$ satisfy the equation.
2. Fractional times also count: at $x = 1.5$, $y = 3.5$.
3. Every time from 0 through 3 has one matching volume.

Answer: All these solution pairs form the entire segment, including endpoints.



PRACTICE 4 problems

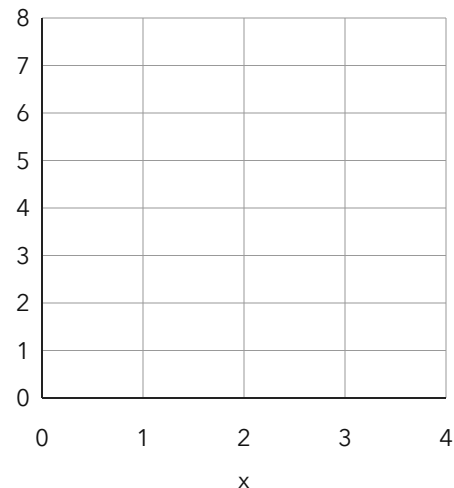
- 1 For $y = x + 3$, decide which of $(1, 4)$ and $(1, 5)$ belongs to the graph. Show substitutions and explain why the other point is excluded.

ANSWER _____

- 2** Water drains according to $y = 6 - x$ liters for $0 \leq x \leq 4$ minutes. Graph every solution in that interval, including fractional times. Explain what $(2.5, 3.5)$ means.

ANSWER _____

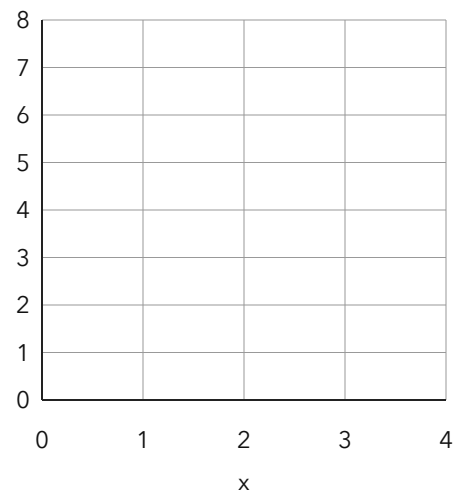
y



- 3** A club gives 1 starter token and 2 tokens for each completed round, up to 3 rounds. Graph all possible (rounds, tokens) pairs. Should you connect them? Explain.

ANSWER _____

y

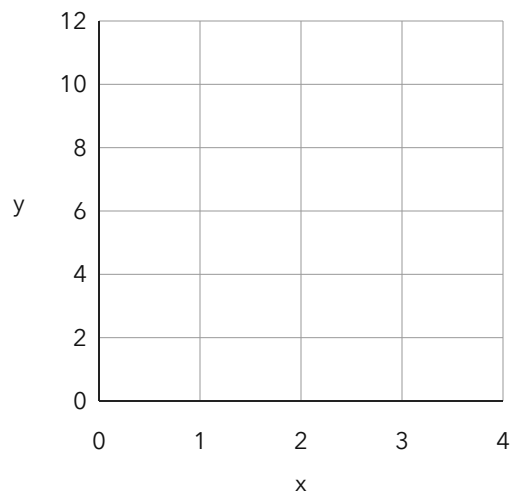


- 4** A different tank follows $y = 3x + 1$ for $0 \leq x \leq 2$ minutes. A student plots only $(0, 1)$, $(1, 4)$, $(2, 7)$ and calls the graph complete. Give one missing solution and explain what else must be drawn.

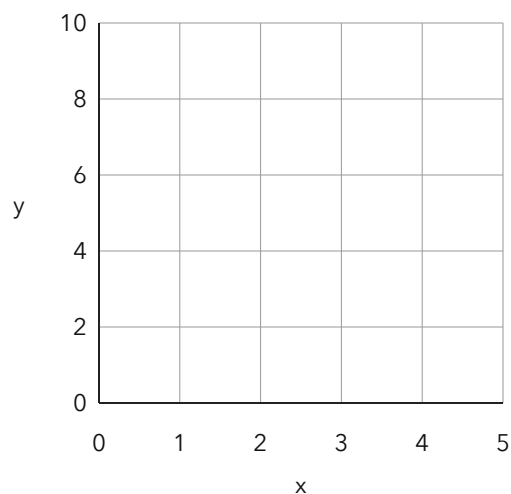
ANSWER _____

APPLY IT Show your work**1**

A ribbon shop charges $y = 3x$ dollars for x meters, allowing any length from 0 to 4 meters. Graph every possible purchase in that interval. Explain why $(2.5, 7.5)$ belongs but $(2.5, 8)$ does not.

REASONING AND ANSWER**2**

A club sells whole notebooks at \$2 each plus a \$1 delivery fee per order. Orders contain 1 to 4 notebooks. Graph the complete set of possible (notebooks, dollars) pairs. Explain why $(1.5, 4)$ is excluded even though $4 = 2(1.5) + 1$.

REASONING AND ANSWER

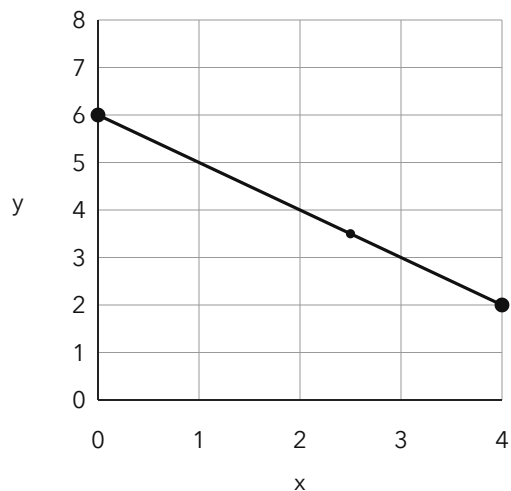
CHALLENGE One step up

A pump has $y = 2x + 1$ liters at any time $0 \leq x \leq 3$ minutes. The tokens in Practice 3 use the same equation. Explain why their complete graphs differ. State what a point must satisfy to belong to each graph.

WORK SPACE AND ANSWER

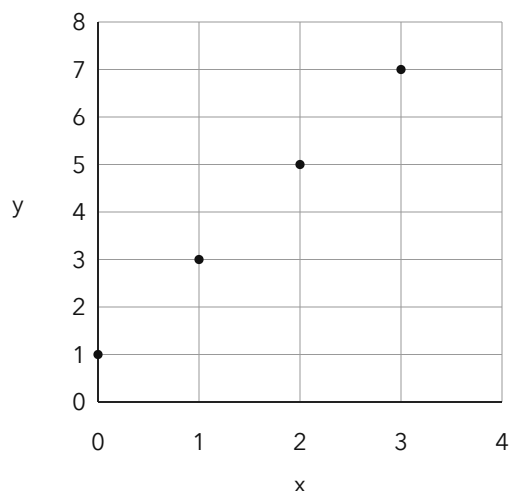
PRACTICE 2

The segment joins $(0, 6)$ to $(4, 2)$. At 2.5 minutes, 3.5 liters remain; $3.5 = 6 - 2.5$.



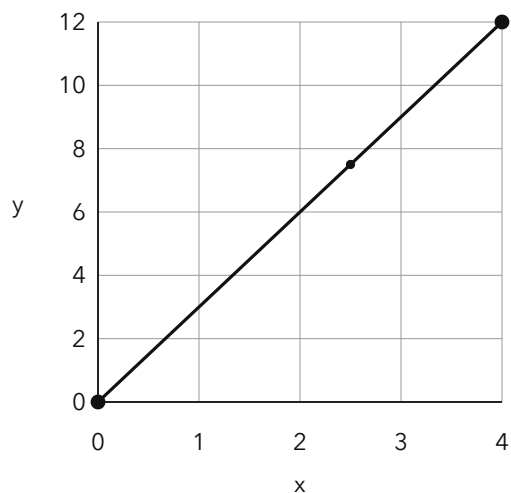
PRACTICE 3

$(0, 1), (1, 3), (2, 5), (3, 7)$. Do not connect: completed rounds are whole-number counts, so fractional-round points are not allowed.



APPLY IT 1

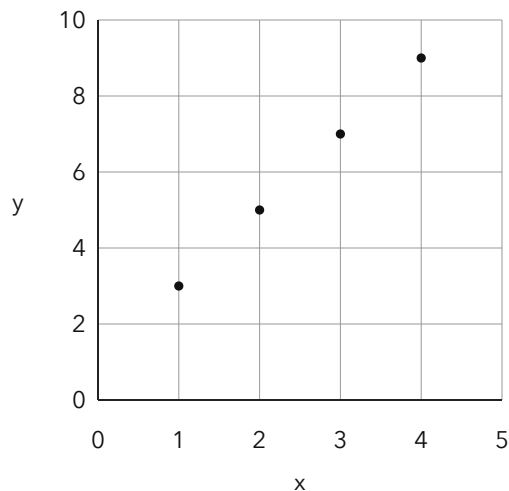
Draw the segment from $(0, 0)$ to $(4, 12)$. $(2.5, 7.5)$ belongs because $3(2.5) = 7.5$; $(2.5, 8)$ does not satisfy the cost equation. Fractional meters are allowed. The segment includes all permitted lengths, not just whole meters.



APPLY IT 2

Plot only $(1, 3), (2, 5), (3, 7), (4, 9)$. A purchase of 1.5 whole notebooks is not allowed, so $(1.5, 4)$ is outside the contextual domain.

Satisfying the equation is necessary, and the input must also satisfy the stated order restriction.



PRACTICE 1

(1, 4) belongs because $4 = 1 + 3$. (1, 5) does not because 5 is not $1 + 3$.

PRACTICE 4

For example (0.5, 2.5), since $2.5 = 3(0.5) + 1$. Draw the entire segment from (0, 1) to (2, 7): every real time from 0 to 2 is allowed. Three dots alone omit infinitely many solutions.

CHALLENGE

The pump permits every real x from 0 to 3, so it has a segment. Tokens permit only $x = 0, 1, 2, 3$, so they have four separate points. In both cases $y = 2x + 1$ must hold; x must also belong to the situation's allowed set.

Require complete contextual solution sets: continuous segments for measured quantities and separate dots for whole-number counts. Endpoints are included. Accept equivalent missing fractional-time solutions and explanations. The excluded fractional notebook point satisfies the algebraic equation but not the situation.