

Polygon Angle Paths

Build interior-angle sums by splitting polygons into triangles.

GRADE 8 • MA.8.GR.1.6

MA.8.GR.1.6 "Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles."

NAME _____

DATE _____

TRIANGLE DECOMPOSITION

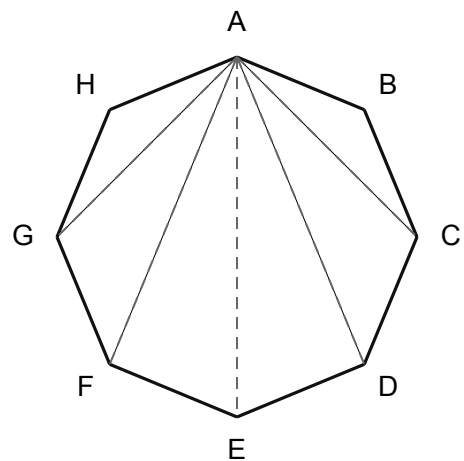
Use a ruler to draw diagonals from one vertex to every nonadjacent vertex. The fan contains $n - 2$ triangles. Each triangle contributes 180° , giving $S = (n - 2) \times 180^\circ$. A regular polygon has equal angles, so each is S/n . Use the sum to solve angle expressions as well as numeric questions.

WORKED EXAMPLE

The regular octagon is split by diagonals from A. Count the triangles and find the interior-angle sum.

1. Count 8 sides, so $n = 8$.
2. The figure can be split into $n - 2 = 8 - 2 = 6$ triangles.
3. Find the sum: $6 \times 180^\circ = 1080^\circ$.

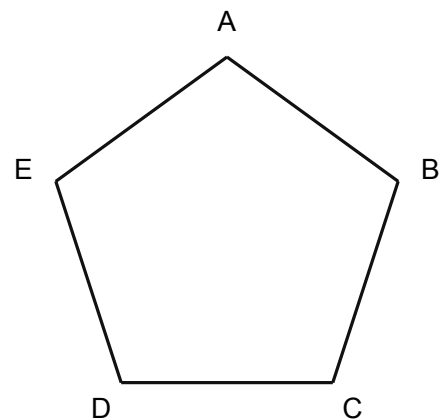
Answer: 1080°



PRACTICE 6 problems

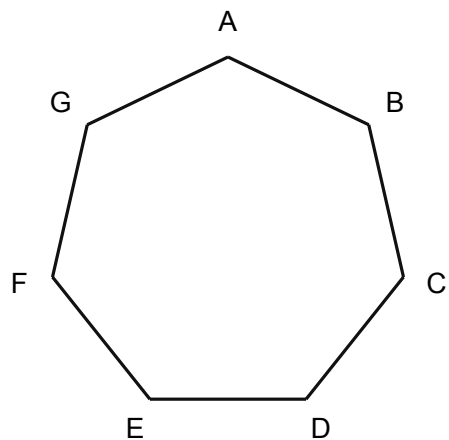
- 1 On the regular pentagon, draw diagonals from A. Count the triangles, then find the interior-angle sum.

ANSWER _____



- 2** On the regular heptagon, draw diagonals from A.
Count the triangles and find the interior-angle sum.

ANSWER _____



- 3** A regular dodecagon is divided into triangles from one vertex. Find the sum of its interior angles.

ANSWER _____

- 4** The sum of the interior angles of a regular polygon is 1260° . How many sides does the polygon have?

ANSWER _____

- 5** Each interior angle of a regular nonagon is $(4x + 20)^\circ$. Find the angle measure and x .

ANSWER _____

- 6** Find the sum of the interior angles of a regular decagon.

ANSWER _____

APPLY IT Show your work

- 1** A gaming club is making a regular 14-sided border for a tournament poster. What is the total measure of the border's interior angles?

WORK SPACE

ANSWER _____

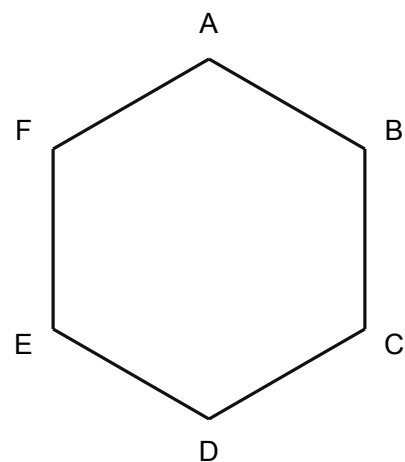
- 2** A game-art token is a regular 18-gon. Each interior angle is $(5x + 10)^\circ$. Find the total interior-angle sum, one angle, and x .

WORK SPACE

CHALLENGE One step up

A student claims a polygon makes n triangles when diagonals are drawn from one vertex. Use vertex B of this regular hexagon to test that claim. Draw the fan, count its triangles, and explain the correct general formula.

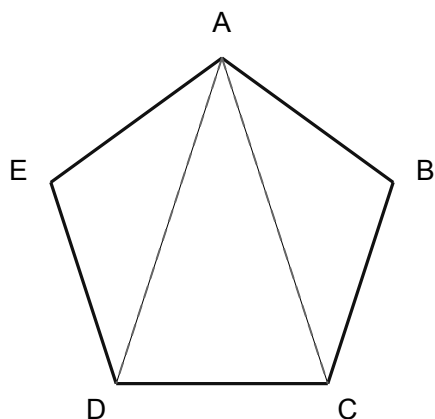
WORK SPACE AND ANSWER



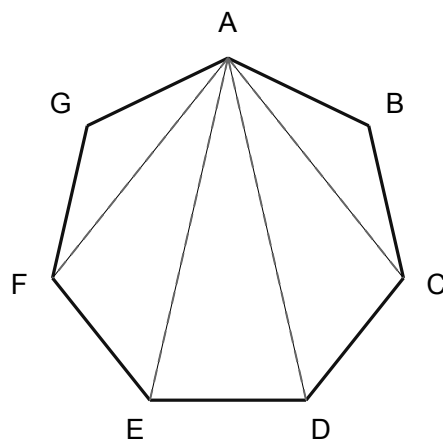
All drawn polygons use equal angular spacing on a circle and are truly regular. Students should draw diagonals, not connect the center to every vertex. Check the fan count and the angle sum. For algebraic angle tasks, distinguish x from the angle measure. Accept equivalent equations and correct fans from the specified vertex.

PRACTICE 1

3 triangles; 540° .

**PRACTICE 2**

5 triangles; 900° .

**PRACTICE 3**

1800°

PRACTICE 4

9 sides

PRACTICE 5

Sum 1260° ; each angle 140° . Thus $4x + 20 = 140$, so $x = 30$.

PRACTICE 6

1440°

APPLY IT 1

2160°

Use $S = (n - 2) * 180^\circ$ with $n = 14$. The sum is $12 * 180^\circ = 2160^\circ$.

APPLY IT 2

Total 2880° ; one angle 160° ; $x = 30$.

$(18 - 2) \times 180^\circ = 2880^\circ$. Divide by 18, then solve $5x + 10 = 160$.

CHALLENGE

The fan from B makes 4 triangles, not 6. In general there are $n - 2$ triangles, so $S = (n - 2) \times 180^\circ$. The two edges adjacent to the vertex are sides, not diagonals.

