

Conditional Chances

Construct a sample space, condition on a group, and test independence.

HIGH SCHOOL • HSS-CP.A.4

CCSS.Math.Content.HSS-CP.A.4 "Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results."

NAME _____

DATE _____

The condition defines the sample space

A two-way frequency table counts each person once in the cell for both recorded categories. Row and column totals are marginal frequencies. A conditional relative frequency divides by the named subgroup's total; compare the same outcome across groups to investigate association. For events A and B with $P(B) > 0$, independence means $P(A | B) = P(A)$, equivalently $P(A \text{ and } B) = P(A)P(B)$.

Worked example

Ten simulated library requests record format and book type. What fraction of print requests are novels?

Practice library requests

Format	Novel	Biography	Total
Print	2	3	5
Digital	1	4	5
Total	3	7	10

1. Restrict attention to the Print row: its total is 5 requests.

2. Use the Novel count in that row: $2/5 = 40\%$.

40% of print requests are novels. This differs from $3/10 = 30\%$ of all requests being novels.

Each record gives grade (9 or 10) and preferred subject when choosing between science and English. These 20 paired records are invented classroom data, not a real school survey. Use them for tasks 1-5. Tasks 6 and 7 supply separate data.

Each ID is one simulated respondent; paired categories stay together

Records	Records	Records	Records
01: Grade 9 / Science	02: Grade 9 / English	03: Grade 10 / English	04: Grade 9 / Science
05: Grade 10 / Science	06: Grade 10 / English	07: Grade 9 / Science	08: Grade 10 / Science
09: Grade 10 / English	10: Grade 9 / Science	11: Grade 10 / Science	12: Grade 10 / English
13: Grade 9 / Science	14: Grade 10 / English	15: Grade 10 / English	16: Grade 9 / Science
17: Grade 10 / English	18: Grade 10 / English	19: Grade 9 / English	20: Grade 10 / English

1. Construct the frequency table

Tally each paired record into the table. Fill all joint frequencies and every row, column and grand total. Check that each respondent is counted exactly once.

Simulated respondent frequencies (complete every cell)

Grade	Science	English	Total
Grade 9			
Grade 10			
Total			

2. Interpret counts

Select one of the 20 records uniformly at random. Find $P(\text{Science})$ and $P(\text{Grade 9 and Science})$. State the event counted by each numerator.

3. Change the denominator

Use your frequency table to complete a row-conditional percentage table. Divide each joint count by its own row total. Each row represents 100% of that row's group; do not add a grand-total row.

Conditional relative frequencies within each row (%) (complete every cell)

Grade	Science	English	Total
Grade 9			
Grade 10			

4. Reverse the condition

Compute $P(\text{Grade 9} \mid \text{Science})$ and compare it with $P(\text{Science} \mid \text{Grade 9})$. Explain why reversing the condition changes the sample space. Give exact fractions.

5. Test independence

For a uniformly selected record, are the events Grade 9 and Science independent? Compare $P(\text{Science} \mid \text{Grade 9})$ with $P(\text{Science})$, then verify using $P(\text{Grade 9 and Science})$ and the product of the marginals.

6. Equal counts, different rates

In a separate simulated survey, Group A has 20 people and 12 club members; Group B has 40 people and 12 club members. Construct the table. A student says membership is equally common because both groups have 12 members. Use conditional percentages to evaluate that claim.

A separate simulated club survey (complete every cell)

Group	Member	Not member	Total
Group A			
Group B			
Total			

7. Build an independent sample space

A box contains 20 tickets: 10 blue and 10 red, with 8 prize tickets altogether. Complete a table in which color and prize status are independent for one uniformly chosen ticket. Show a conditional-probability check and a product check.

Simulated prize tickets (complete every cell)

Color	Prize	No prize	Total
Blue			
Red			
Total			

TEACHER NOTES

Require both category labels, all joint counts and margins. Keep paired observations intact. Row-conditional tables use the row totals, not the overall total. Accept equivalent fractions and accurate explanations; percentages here are exact. Simulated classroom data describe only the supplied records. For a uniformly selected record, empirical probabilities can test independence within that sample space; they do not prove independence or causation in a wider population.

TASK 1

The completed table below counts every paired record once; both sets of marginal totals sum to the number of records.

Simulated respondent frequencies

GRADE	SCIENCE	ENGLISH	TOTAL
GRADE 9	6	2	8
GRADE 10	3	9	12
TOTAL	9	11	20

TASK 2

$P(\text{Science}) = 9/20 = 0.45$; its numerator counts all science-preferring records. $P(\text{Grade 9 and Science}) = 6/20 = 0.30$; its numerator counts only records in both categories.

TASK 3

Each row uses its own group total as the denominator. The two percentages within a row sum to 100%.

Conditional relative frequencies within each row (%)

GRADE	SCIENCE	ENGLISH	TOTAL
GRADE 9	75%	25%	100%
GRADE 10	25%	75%	100%

TASK 4

$P(\text{Grade 9} \mid \text{Science}) = 6/9 = 2/3$; $P(\text{Science} \mid \text{Grade 9}) = 6/8 = 3/4$. Conditioning restricts the sample space to 9 science-preferring records or 8 Grade 9 records, respectively. Name the starting group before choosing a denominator; the joint cell stays the same when the condition is reversed.

TASK 5

They are dependent in this empirical sample space: $P(\text{Science} \mid \text{Grade 9}) = 6/8 = 0.75$ differs from $P(\text{Science}) = 9/20 = 0.45$. Also $P(\text{Grade 9 and Science}) = 6/20 = 0.30$, while $(8/20)(9/20) = 0.18$. Compare rates that use the same outcome and explicitly name each denominator.

TASK 6

The claim is false: Group A has $12/20 = 60\%$ members, while Group B has $12/40 = 30\%$. Equal joint counts do not imply equal within-group rates when group sizes differ. Subtract each club count from its group total to find nonmembers, then divide club counts by the corresponding group totals.

A separate simulated club survey

GROUP	MEMBER	NOT MEMBER	TOTAL
GROUP A	12	8	20
GROUP B	12	28	40
TOTAL	24	36	60

TASK 7

Each color has 4 prize and 6 no-prize tickets. $P(\text{Prize} \mid \text{Blue}) = 4/10 = 0.40 = P(\text{Prize}) = 8/20$. $P(\text{Blue and Prize}) = 4/20 = 0.20 = (10/20)(8/20)$. Thus color and prize status are independent in this specified sample space.

Simulated prize tickets

COLOR	PRIZE	NO PRIZE	TOTAL
BLUE	4	6	10
RED	4	6	10
TOTAL	8	12	20