

Unit Circle Moves

Use coordinates and right triangles to find six trig values.

HIGH SCHOOL • MA.912.T.2.2

MA.912.T.2.2 "Define the six basic trigonometric functions for all real numbers by identifying corresponding angle measures and using right triangles drawn in the unit circle."

NAME _____

DATE _____

You will need: Pencil

UNIT CIRCLE RULES

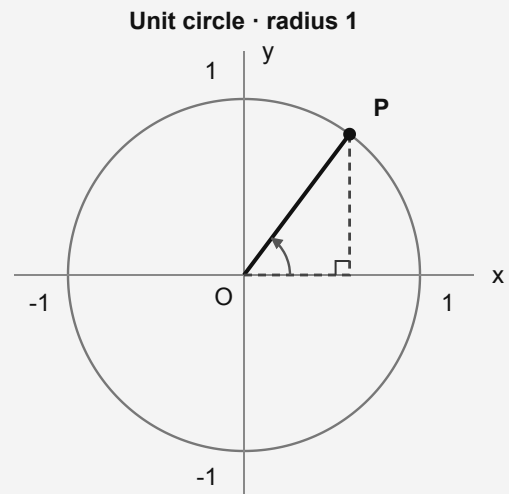
For any real angle θ , its terminal point on the unit circle is (x, y) , where $\cos \theta = x$ and $\sin \theta = y$. Then $\tan \theta = y/x$, $\csc \theta = 1/y$, $\sec \theta = 1/x$, and $\cot \theta = x/y$. Angles differing by 360° or 2π radians share the same point, and a function is undefined when its denominator is 0. The perpendicular from the terminal point to the x-axis forms a reference right triangle. Use its side lengths and the coordinate signs; an axis angle has a zero-length leg. Give exact values or write undefined in all answers.

WORKED EXAMPLE

In the unit-circle diagram, $P = (3/5, 4/5)$. Find all six trigonometric functions of θ .

1. $\cos \theta = 3/5$ and $\sin \theta = 4/5$.
2. $\tan \theta = (4/5)/(3/5) = 4/3$.
3. $\csc \theta = 5/4$, $\sec \theta = 5/3$, and $\cot \theta = 3/4$.

Answer: $\sin \theta = 4/5$, $\cos \theta = 3/5$, $\tan \theta = 4/3$, $\csc \theta = 5/4$,
 $\sec \theta = 5/3$, $\cot \theta = 3/4$

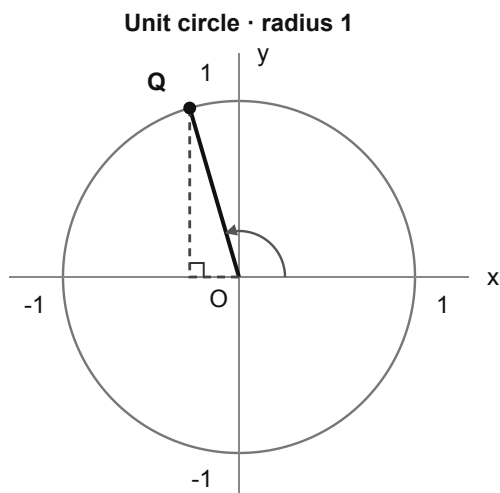


θ : counterclockwise from $+x$

Arc direction: counterclockwise

PRACTICE 4 problems

- 1** Point $Q = (-7/25, 24/25)$ lies on the unit circle in Quadrant II. Find $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$.



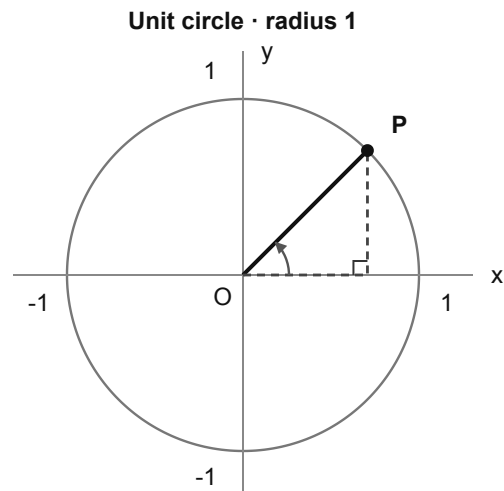
θ : counterclockwise from $+x$
Arc direction: counterclockwise

sin _____ **cos** _____

tan _____ **csc** _____

sec _____ **cot** _____

- 2** $\theta = 45^\circ$ and $\varphi = 405^\circ$ are coterminal. Find $\sin \varphi$, $\cos \varphi$, $\tan \varphi$, $\csc \varphi$, $\sec \varphi$, and $\cot \varphi$.



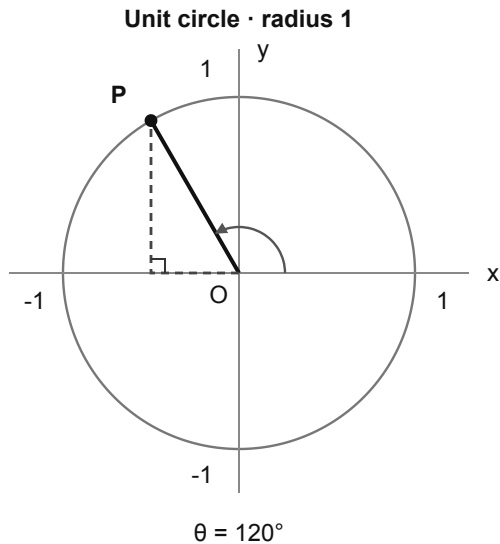
$\varphi = 405^\circ$; same ray as 45°
Arc direction: counterclockwise

sin _____ **cos** _____

tan _____ **csc** _____

sec _____ **cot** _____

- 3** For $\theta = 120^\circ$, fill all six blanks.



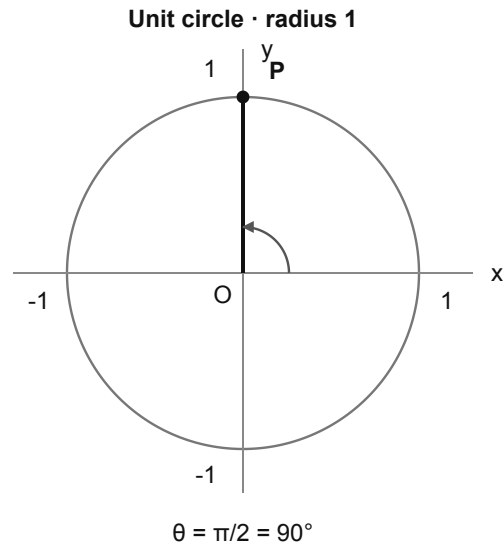
Arc direction: counterclockwise

sin _____ **cos** _____

tan _____ **csc** _____

sec _____ **cot** _____

- 4** For $\theta = \pi/2$, fill all six blanks.



Arc direction: counterclockwise

sin _____ **cos** _____

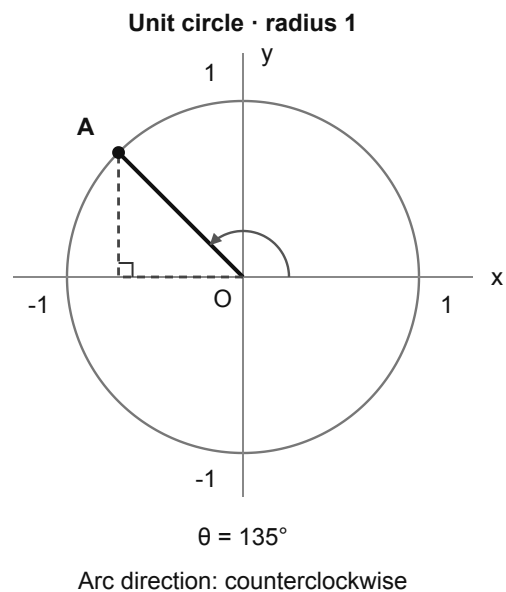
tan _____ **csc** _____

sec _____ **cot** _____

APPLY IT Show your work

- 1** A game designer places sprite A at $\theta = 135^\circ$ on the unit circle. Use the diagram to find its x-coordinate, y-coordinate, and $\tan \theta$.

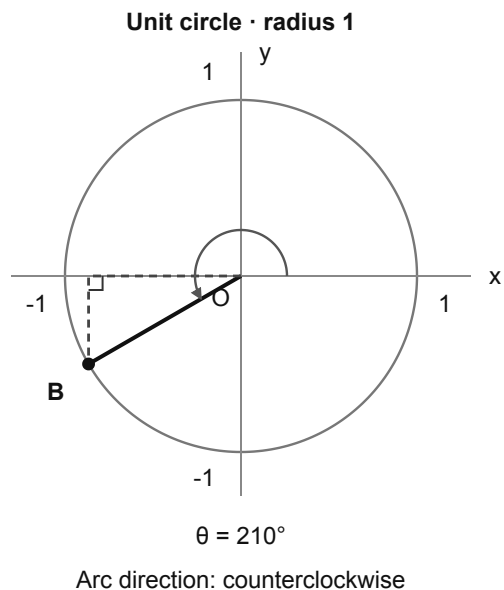
ANSWER _____



2

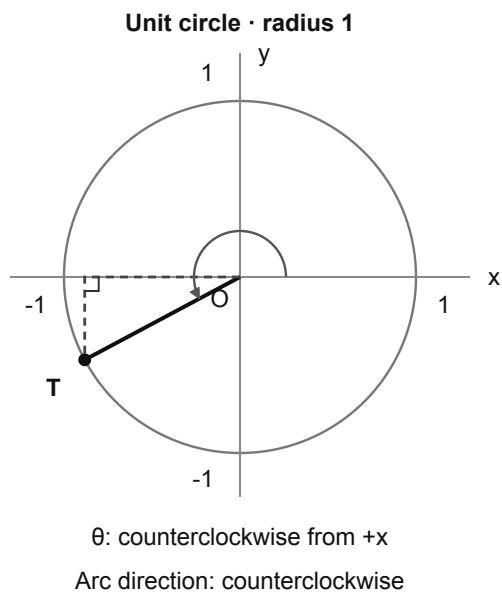
A spotlight points along OB at $\theta = 210^\circ$ on the unit circle. Use the diagram to find $\cos \theta$ and $\sec \theta$.

ANSWER _____



CHALLENGE One step up

If $\sin \theta = -8/17$ and θ is in Quadrant III, explain how rotating the terminal ray 180° changes $\sin \theta$, $\cos \theta$, and $\tan \theta$. Give all three values at θ and at $\theta + \pi$.



PRACTICE 1

Point Q = $(-7/25, 24/25)$ lies on the unit circle in Quadrant II. Find $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$.

$$\sin \theta = 24/25, \cos \theta = -7/25, \tan \theta = -24/7, \csc \theta = 25/24, \sec \theta = -25/7, \cot \theta = -7/24$$

PRACTICE 2

$\theta = 45^\circ$ and $\phi = 405^\circ$ are coterminal. Find $\sin \phi$, $\cos \phi$, $\tan \phi$, $\csc \phi$, $\sec \phi$, and $\cot \phi$.

$$\sin \phi = \sqrt{2}/2, \cos \phi = \sqrt{2}/2, \tan \phi = 1, \csc \phi = \sqrt{2}, \sec \phi = \sqrt{2}, \cot \phi = 1$$

PRACTICE 3

For $\theta = 120^\circ$, fill all six blanks.

$$\sin \theta = \sqrt{3}/2, \cos \theta = -1/2, \tan \theta = -\sqrt{3}, \csc \theta = 2\sqrt{3}/3, \sec \theta = -2, \cot \theta = -\sqrt{3}/3$$

PRACTICE 4

For $\theta = \pi/2$, fill all six blanks.

$$\sin \theta = 1, \cos \theta = 0, \tan \theta = \text{undefined}, \csc \theta = 1, \sec \theta = \text{undefined}, \cot \theta = 0$$

APPLY IT 1

$x = -\sqrt{2}/2$, $y = \sqrt{2}/2$, $\tan \theta = -1$ At 135° , the point is in Quadrant II with a 45° reference angle. Its unit-circle coordinates are $(-\sqrt{2}/2, \sqrt{2}/2)$, so $y/x = -1$.

APPLY IT 2

$\cos \theta = -\sqrt{3}/2$, $\sec \theta = -2\sqrt{3}/3$ The 210° ray has a 30° reference angle in Quadrant III. The x-coordinate is $-\sqrt{3}/2$, and its reciprocal is $-2\sqrt{3}/3$.

CHALLENGE

At θ : $\sin \theta = -8/17$, $\cos \theta = -15/17$, $\tan \theta = 8/15$. At $\theta + \pi$: $\sin(\theta + \pi) = 8/17$, $\cos(\theta + \pi) = 15/17$, $\tan(\theta + \pi) = 8/15$. A 180° rotation changes both coordinates to their opposites, so sine and cosine change signs while tangent stays the same.

Accept equivalent exact forms, including unrationalized reciprocal forms such as $-2/\sqrt{3}$. Accept "undefined" or "does not exist" for functions with a zero denominator. The six labeled practice blanks correspond to the six requested functions. Reference-triangle legs are distances; coordinate signs come from the quadrant. For $\theta = \pi/2$, the terminal point is $(0,1)$, so tan and sec are undefined. In the challenge, T' is opposite T ; the dashed teacher ray shows $\theta + \pi$ modulo a full turn.

