LESSON MAKER PRO

Binomial Power Moves

Expand binomials with Pascal's Triangle patterns.

HIGH SCHOOL · HSA-APR.C.5

CCSS.Math.Content.HSA-APR.C.5 "(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle."

NAME _____ DATE _____

THE EXPANSION PATTERN

To expand a binomial raised to a positive whole-number power, use the matching row of Pascal's Triangle for the coefficients. Write terms in order so the exponent on the first term decreases by 1 each time while the exponent on the second term increases by 1 each time.

WORKED EXAMPLE

Use the n=3 Pascal row to expand$(x+7)^3$. Explain why the x^2 coefficient is3×7.

n	Pascal coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

1. Choose the7from exactly one of three factors; the three choices produce the coefficient3.
2. Use the Pascal's Triangle coefficients 1, 3, 3, 1.
3. Write $x^3 + 3x^2(7) + 3x(7)^2 + 7^3$.
4. Simplify each term and combine the result.

Answer: $x^3 + 21x^2 + 147x + 343$

PRACTICE 6 problems

1 Expand $(x + 1)^4$.

ANSWER _____

2 Expand $(2x + 1)^4$.

ANSWER _____

3 Expand $(x - 1)^5$.

ANSWER _____

4 What is the coefficient of x^2 in $(x + 1)^4$?

ANSWER _____

5 In$(x+5)^4$, find the x^3 coefficient by counting which one of the four factors contributes5. Show why there are four choices.

Chosen constant factor	Contribution to x^3
1	_____
2	_____
3	_____
4	_____

ANSWER _____

_____

6 Use the n=2 Pascal row to expand$(x+4)^2$. Explain the middle coefficient.

Choose from first factor	Choose from second	Product
x	x	_____
x	4	_____
4	x	_____
4	4	_____

ANSWER _____

_____

APPLY IT Show your work

1 A game designer models the score effect of four upgrade slots with $(x + 2)^4$, where x is a base score factor. Expand the expression to show every term in the model.

2 A music producer models an audio filter's five-stage intensity adjustment with $(2x - 1)^5$. Expand the expression to show the full adjustment model.

CHALLENGE One step up

Without fully expanding $(x + 1)^8$, find and explain the coefficient of x^2.

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Expand $(x + 1)^4$.	$\mathbf{x^4 + 4x^3 + 6x^2 + 4x + 1}$
2	Expand $(2x + 1)^4$.	$\mathbf{16x^4 + 32x^3 + 24x^2 + 8x + 1}$
3	Expand $(x - 1)^5$.	$\mathbf{x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1}$
4	What is the coefficient of x^2 in $(x + 1)^4$?	**6**
5	In$(x+5)^4$, find the x^3 coefficient by counting which one of the four factors contributes5. Show why there are four choices.	**20**
6	Use the n=2 Pascal row to expand$(x+4)^2$. Explain the middle coefficient.	$\mathbf{x^2 + 8x + 16}$

PART 2 · APPLY IT

1. $\mathbf{x^4 + 8x^3 + 24x^2 + 32x + 16}$ Use Pascal's Triangle coefficients 1, 4, 6, 4, 1. Multiply each coefficient by the appropriate power of 2 as the power of x decreases.
2. $\mathbf{32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1}$ Use coefficients 1, 5, 10, 10, 5, 1 and substitute 2x for the first term and -1 for the second term. The signs alternate because the second term is negative.

CHALLENGE

28. The x^2 term comes from choosing x from exactly 2 of the 8 factors, so its coefficient is C(8, 2) = 28.

Accept equivalent expanded forms with terms written in any order. Coefficients and signs must be correct, and expressions should be simplified. Coefficients count choices of factors, not the area of an unrelated integer array. For practice5:4×5=20; practice6 middle term2×4x=8x.