LESSON MAKER PRO

Around Again

Use unit-circle turns to find sine and cosine.

HIGH SCHOOL · HSF-TF.A.2

CCSS.Math.Content.HSF-TF.A.2 "Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle."

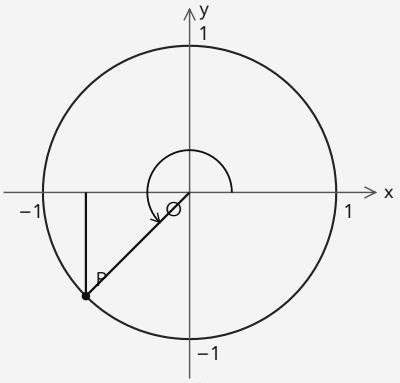
NAME _____ DATE _____

ONE CIRCLE, ANY ANGLE

On the unit circle, the terminal point for an angle θ is (cos θ, sin θ). Radians measure rotation around the circle, and one full counterclockwise turn is 2π radians. Adding or subtracting 2π keeps the same terminal point.

WORKED EXAMPLE

On the unit circle, use the angle shown to find its radian measure, terminal point, cosine, and sine.



225°; radius = 1

1. 225° × π/180° = 5π/4.
2. The angle is 45° past π in Quadrant III.
3. The terminal point is (-√2/2, -√2/2).

Answer: 5π/4; terminal point (-√2/2, -√2/2); cos θ = -√2/2; sin θ = -√2/2

PRACTICE 4 problems

1 Find the terminal point for θ = -π/2 on the unit circle.

ANSWER _____

2 For θ = 13π/6, find the coterminal angle in [0, 2π) and the terminal point.

ANSWER _____

3 A terminal point is (0, -1). Give its angle in the interval [-2π, 0].

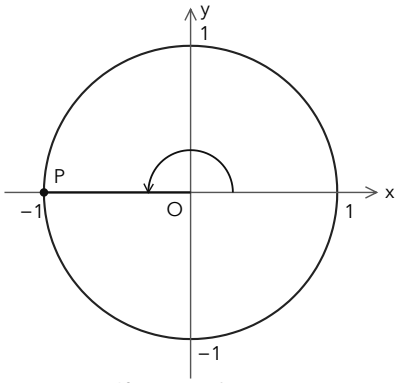
ANSWER _____

4 Find cos θ and sin θ for θ = 7π/3.

ANSWER _____

APPLY IT Show your work

1 A game designer places a sprite at P=(-1,0) on the unit circle shown. The sprite rotates counterclockwise from (1,0) and stops when it first reaches P. What is the rotation in radians, and what are cos θ and sin θ?

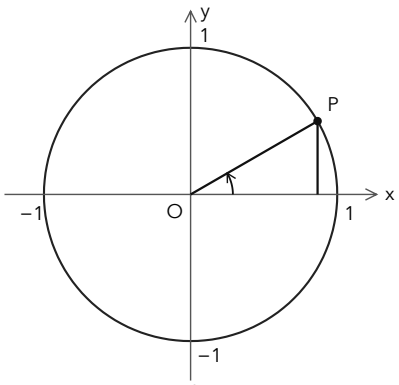


Half-turn; radius = 1

2 A music app's turntable marker starts at (1, 0) and rotates 17π/6 radians counterclockwise. How many complete rotations occur, and where does the marker end?

CHALLENGE One step up

The figure shows a terminal side on the unit circle. Explain why $\sin(\theta + 2\pi k)$ and $\cos(\theta + 2\pi k)$ have the same values as $\sin \theta$ and $\cos \theta$ for every integer k. Include the sine and cosine values for the shown angle. Explain how any real input t defines a terminal point: use signed arc length on a radius-1 circle, with positive motion counterclockwise and negative motion clockwise.



30°; radius = 1

ANSWER KEY

Around Again

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the terminal point for θ = -π/2 on the unit circle.	**(0, -1)**
2	For θ = 13π/6, find the coterminal angle in [0, 2π) and the terminal point.	**π/6; (√3/2, 1/2)**
3	A terminal point is (0, -1). Give its angle in the interval [-2π, 0].	**-π/2**
4	Find cos θ and sin θ for θ = 7π/3.	**cos θ = 1/2; sin θ = √3/2**

PART 2 · APPLY IT

1. **π; cos θ = -1; sin θ = 0** Point P is at (-1, 0), halfway around the unit circle from (1, 0). Half of 2π is π.
2. **1 complete rotation; terminal point (-√3/2, 1/2)** 17π/6 = 2π + 5π/6, so one full turn is removed. The terminal point for 5π/6 is (-√3/2, 1/2).

CHALLENGE

The shown 30° angle is π/6, so cos θ = √3/2 and sin θ = 1/2. Adding 2πk adds whole turns, which return the terminal side to the same point, so the cosine and sine values stay √3/2 and 1/2. Any real t specifies a signed arc length; walking that distance in the stated direction determines a terminal point even after multiple turns. The coordinates define cosine and sine for every real input.

Accept equivalent radical forms and ordered-pair formatting. For the challenge, accept any explanation that states that whole 2π turns return to the same terminal point.