LESSON MAKER PRO

Focus Locus Lab

Build ellipse and hyperbola equations from their foci.

HIGH SCHOOL · HSG-GPE.A.3

CCSS.Math.Content.HSG-GPE.A.3 "(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant."

NAME _____ DATE _____

DISTANCE RULES

For a point P, write its distances to both foci using the distance formula. An ellipse has a constant distance sum; a hyperbola has a constant absolute distance difference. Isolate radicals and square carefully to derive an equation, then check the branch/sign conditions.

WORKED EXAMPLE

Derive the ellipse with foci $(-2,0),(2,0)$ and distance sum 8. Let $L=\sqrt{((x+2)^2+y^2)}$, $R=\sqrt{((x-2)^2+y^2)}$.

1. $L+R=8$ and $L^2-R^2=8x$, so $L-R=x$. Thus $L=4+x/2$.
2. Square: $(x+2)^2+y^2=(4+x/2)^2$.
3. Expand and simplify: $3x^2/4+y^2=12$. Divide by 12.
4. The resulting ellipse has $-4\le x\le 4$, so $L=4+x/2$ and $R=4-x/2$ stay nonnegative and sum to 8.

Answer: $x^2/16+y^2/12=1$

PRACTICE 5 problems

1 Use the focal relationship to find the equation of the ellipse with foci at (0, -1) and (0, 1). The sum of the distances is 6.

ANSWER _____

2 Use the focal relationship to find the equation of the hyperbola with foci at (-5, 0) and (5, 0). The positive difference of the distances is 6.

ANSWER _____

3 Use the focal relationship to find the equation of the hyperbola with foci at (0, -7) and (0, 7). The positive difference of the distances is 10.

ANSWER _____

4 Use the focal relationship to find the equation of the ellipse with foci at (6, -3) and (6, 3). The sum of the distances is 14.

ANSWER _____

5 Use the focal relationship to find the equation of the hyperbola with foci at (1, -6) and (1, 6). The positive difference of the distances is 10.

ANSWER _____

APPLY IT Show your work

1 For a drone light show, signal beacons are at (5, 1) and (5, 11). The app traces all locations whose distances from the beacons add to 18 units. Find the equation of the path.

2 Two beacons are at (−9,1) and (9,1). Their absolute distance difference is 14. Derive the equation, starting from $L=\sqrt{((x+9)^2+(y-1)^2)}$ and $R=\sqrt{((x-9)^2+(y-1)^2)}$. On the right branch use $L-R=14$ and $(L-R)(L+R)=36x$. Find L, square, simplify, and state how the left branch is included.

CHALLENGE One step up

An ellipse has foci at (-7, 1) and (5, 1), and the point (6, 1) lies on the ellipse. Find its equation. Then state why the major axis is horizontal.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the focal relationship to find the equation of the ellipse with foci at (0, -1) and (0, 1). The sum of the distances is 6.	**$x^2/8 + y^2/9 = 1$**
2	Use the focal relationship to find the equation of the hyperbola with foci at (-5, 0) and (5, 0). The positive difference of the distances is 6.	**$x^2/9 - y^2/16 = 1$**
3	Use the focal relationship to find the equation of the hyperbola with foci at (0, -7) and (0, 7). The positive difference of the distances is 10.	**$y^2/25 - x^2/24 = 1$**
4	Use the focal relationship to find the equation of the ellipse with foci at (6, -3) and (6, 3). The sum of the distances is 14.	**$(x - 6)^2/40 + y^2/49 = 1$**
5	Use the focal relationship to find the equation of the hyperbola with foci at (1, -6) and (1, 6). The positive difference of the distances is 10.	**$y^2/25 - (x - 1)^2/11 = 1$**

PART 2 · APPLY IT

1. **$(x - 5)^2/56 + (y - 6)^2/81 = 1$** The center is (5, 6), with a = 9 and c = 5. Then $b^2 = 81 - 25 = 56$, and the major axis is vertical.
2. **$L+R=18x/7$, so $L=9x/7+7$. Squaring gives $(x+9)^2+(y-1)^2=(9x/7+7)^2$, hence $32x^2/49-(y-1)^2=32$. Thus $x^2/49-(y-1)^2/32=1$. Right branch $x\geq 7$; left branch $x\leq -7$ follows by reflection and $R-L=14$.** Require the distance equation and algebra, not just substitution into a remembered conic formula. Absolute difference includes both branches.

CHALLENGE

$(x + 1)^2/49 + (y - 1)^2/13 = 1$. The foci have the same y-coordinate, so the major axis is horizontal.

Accept equivalent equations with terms in either order. Require correctly shifted squared terms when the center is not the origin. For the worked ellipse and Apply 2 require the displayed distance-based derivation. Check sign conditions after squaring; the two hyperbola branches represent the absolute difference.