LESSON MAKER PRO

Approach From One Side

Read the behavior from the left or right.

HIGH SCHOOL · MA.912.C.1.4

MA.912.C.1.4 "Find one-sided limits."

NAME _____ DATE _____

ONE SIDE AT A TIME

A left-hand limit looks at x-values less than the target value. A right-hand limit looks at x-values greater than the target value. Use only the rule, pattern, or graph behavior from the requested side.

WORKED EXAMPLE

Find $\lim_{x \to 7^-} (x^2 - 49)/(x - 7)$.

1. Factor the numerator: $x^2 - 49 = (x - 7)(x + 7)$.
2. For x not equal to 7, the expression simplifies to $x + 7$.
3. As x approaches 7 from the left, $x + 7$ approaches 14.

Answer: 14

PRACTICE 6 problems

1 For f(x) = x + 5 if x < 3, and f(x) = 4x - 1 if x >= 3, find lim x->3- f(x).

ANSWER _____

2 For g(x) = 6 - x if x <= -4, and g(x) = x + 9 if x > -4, find lim x->-4+ g(x).

ANSWER _____

3 Find lim x->0- |x|/x.

ANSWER _____

4 Find lim x->-3+ ((x + 3)(x - 5))/(x + 3).

ANSWER _____

5 Let p(x) be the greatest integer less than or equal to x. Find lim x->5- p(x).

ANSWER _____

6 Find lim x->6- 1/(x - 6).

ANSWER _____

APPLY IT Show your work

1 A game gives bonus points B(x), where x is elapsed time in seconds, allowed to take any real value from 0 onward. B(x) = x + 9 for x < 10, and B(x) = 3x - 11 for x >= 10. Find lim x->10- B(x). What does this mean in context?

WORK SPACE

ANSWER

2 An online art contest displays a featured score R(t), where t is the number of hours after entries open. R(t) = 5t - 8 for t < 4, and R(t) = 3t + 3 for t >= 4. Find lim t->4+ R(t). What does this mean in context?

WORK SPACE

ANSWER

CHALLENGE One step up

For h(x) = ((x - 5)(x + 4))/(x - 5) if x < 5, and h(x) = 3x - 6 if x >= 5, find lim x->5- h(x), lim x->5+ h(x), and lim x->5 h(x). Explain how you know the two-sided limit exists.

WORK SPACE AND ANSWER

ANSWER KEY

Approach From One Side

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x) = x + 5 if x < 3, and f(x) = 4x - 1 if x >= 3, find lim x->3- f(x).	**8**
2	For g(x) = 6 - x if x <= -4, and g(x) = x + 9 if x > -4, find lim x->-4+ g(x).	**5**
3	Find lim x->0- |x|/x.	**-1**
4	Find lim x->-3+ ((x + 3)(x - 5))/(x + 3).	**-8**
5	Let p(x) be the greatest integer less than or equal to x. Find lim x->5- p(x).	**4**
6	Find lim x->6- 1/(x - 6).	**-infinity**

PART 2 · APPLY IT

1. **19** as elapsed time approaches 10 seconds from below, the bonus points get closer to 19. Use the x < 10 rule because the score approaches 10 from the left. Substitute 10 into x + 9.
2. **15** just after 4 hours, the featured score gets closer to 15. Use the t >= 4 rule because time approaches 4 from the right. Substitute 4 into 3t + 3.

CHALLENGE

lim x->5- h(x) = 9, lim x->5+ h(x) = 9, and lim x->5 h(x) = 9 because the one-sided limits are equal.

Accept equivalent limit notation, including -∞ for -infinity. In the challenge, require both one-sided limits and the conclusion that the two-sided limit exists because they match.