LESSON MAKER PRO

Tangent Slope Quest

Use derivatives to find instant slopes, including vertical tangents.

HIGH SCHOOL · MA.912.C.3.1

MA.912.C.3.1 "Find the slope of a curve at a point, including points at which there are vertical tangent lines."

NAME _____ DATE _____

SLOPE AT ONE POINT

Where f′(a) exists, it gives the tangent slope. An undefined derivative alone does not prove a vertical tangent: the point must be on the curve and the tangent direction must approach vertical. For a smooth implicit curve, a regular parametrization with dx/ds = 0 and dy/ds ≠ 0 identifies a vertical tangent.

WORKED EXAMPLE

Find the slope of $y=12x^2+17x$ at $x=19$.

1. Differentiate: $y'=24x+17$.
2. Substitute $x=19$: $y'(19)=24(19)+17$.
3. Simplify: $456+17=473$.

Answer: 473

PRACTICE 6 problems

1 Find the slope of $p(x)=6x^2+8x-9$ at $x=-2$.

ANSWER _____

2 Find the slope of $q(x)=9x^2-5x+1$ at $x=-1$.

ANSWER _____

3 Find the slope of $r(x)=1/x$ at $x=-5$.

ANSWER _____

4 Find the slope of $s(x)=x^{1/2}$ at $x=9$.

ANSWER _____

5 The curve $x^2+9y^2=36$ passes through (6,0). Find dy/dx at this point. State whether the tangent is vertical.

ANSWER _____

6 The curve $x=y^5+2$ passes through (2,0). Find dy/dx at this point. State whether the tangent is vertical.

ANSWER _____

APPLY IT Show your work

1 A phone battery's charge is modeled by $B(t)=100-2t^2$, where B is the battery percentage and t is time in hours. Find the slope of the curve at t=5. What does the slope mean in this situation?

WORK SPACE

ANSWER

2 A student coding a racing game models one sharp section of a track by $x=y^5+8$, where x and y are coordinates in meters. At (8,0), find dy/dx and describe the tangent line.

WORK SPACE

ANSWER

CHALLENGE One step up

The curve $x^5+y^5=32$ has a vertical tangent. Find every point on the curve where the tangent is vertical. Explain how you know there are no other vertical tangents.

WORK SPACE AND ANSWER

ANSWER KEY

Tangent Slope Quest

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the slope of $p(x)=6x^2+8x-9$ at $x=-2$.	**-16**
2	Find the slope of $q(x)=9x^2-5x+1$ at $x=-1$.	**-23**
3	Find the slope of $r(x)=1/x$ at $x=-5$.	**-1/25**
4	Find the slope of $s(x)=x^{1/2}$ at $x=9$.	**1/6**
5	The curve $x^2+9y^2=36$ passes through (6,0). Find dy/dx at this point. State whether the tangent is vertical.	**The slope is undefined. The tangent is vertical.**
6	The curve $x=y^5+2$ passes through (2,0). Find dy/dx at this point. State whether the tangent is vertical.	**The slope is undefined. The tangent is vertical.**

PART 2 · APPLY IT

1. **-20 percentage points per hour. At 5 hours, the battery charge is decreasing at 20 percentage points per hour.** Differentiate to get $B'(t)=-4t$. Substitute $t=5$ to get $B'(5)=-20$.
2. **dy/dx is undefined, and the tangent line is vertical.** Use y as a parameter: $(x, y) = (y^5 + 8, y)$. At $y = 0$, the tangent vector is $(5y^4, 1) = (0, 1)$, so the tangent is the vertical line $x = 8$. The reciprocal derivative $1/(5y^4)$ has no finite value there.

CHALLENGE

The only point is (2,0). Implicit differentiation gives $dy/dx=-x^4/y^4$, which is undefined only when $y=0$ and x is not 0. Substituting $y=0$ into $x^5+y^5=32$ gives $x=2$. The gradient $(5x^4, 5y^4)$ is nonzero everywhere on this level curve. Its tangent vector $(5y^4, -5x^4)$ is vertical exactly when $y = 0$, establishing the claim.

Accept equivalent fraction forms for numerical slopes. For vertical tangents, accept "undefined slope," "no finite slope," or "DNE" only when the student also identifies the tangent as vertical.