LESSON MAKER PRO

Derivative Direction Detectives

Use derivative signs to track where a function rises or falls.

HIGH SCHOOL · MA.912.C.3.3

MA.912.C.3.3 "Determine where a function is decreasing and increasing using its derivative."

NAME _____ **DATE** _____

READ THE DERIVATIVE SIGN

First find f'(x). Mark values in the domain where f'(x) = 0 or where f'(x) is undefined, then test the sign of f'(x) in each interval. The function is increasing where f'(x) is positive and decreasing where f'(x) is negative.

WORKED EXAMPLE

Determine where $h(x) = x^2 - 10x$ is increasing and decreasing.

1. Find the derivative: $h'(x) = 2x - 10$.
2. Set the derivative equal to zero: $2x - 10 = 0$, so $x = 5$.
3. For $x < 5$, $h'(x) < 0$. For $x > 5$, $h'(x) > 0$.
4. A negative derivative means decreasing, and a positive derivative means increasing.

Answer: h(x) is decreasing on $(-\infty, 5)$ and increasing on $(5, \infty)$.

PRACTICE 6 problems

1 Determine where $f(x) = x^2 + 8x$ is increasing and decreasing.

ANSWER _____

2 Determine where $f(x) = -x^2 + 14x$ is increasing and decreasing.

ANSWER _____

3 Determine where $f(x) = x^3 - 27x$ is increasing and decreasing.

ANSWER _____

4 Determine where $f(x) = x^3 + 6x^2$ is increasing and decreasing.

ANSWER _____

5 A differentiable function has $f'(x) = (x - 1)(x + 9)$. Determine where f is increasing and decreasing.

ANSWER _____

6 A differentiable function has $f'(x) = -(x + 6)(x - 8)$. Determine where f is increasing and decreasing.

ANSWER _____

APPLY IT Show your work

1 A student club models its profit from selling batches of custom stickers by $P(x) = -x^2 + 18x - 32$, where x is the number of batches and $x \geq 0$. During what intervals is profit increasing and decreasing?

WORK SPACE

ANSWER

2 The number of active viewers for a livestream is modeled by $V(x) = -x^2 + 16x + 40$, where x is hours after the stream begins and $0 \leq x \leq 18$. During what intervals is the viewer count increasing and decreasing?

WORK SPACE

ANSWER

CHALLENGE One step up

For $f(x) = x^3/(x^2 - 1)$, determine where f is increasing and decreasing. Explain why $x = -1$ and $x = 1$ must split your intervals.

WORK SPACE AND ANSWER

ANSWER KEY

Derivative Direction Detectives

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Determine where $f(x) = x^2 + 8x$ is increasing and decreasing.	**Decreasing on $(-\infty, -4)$; increasing on $(-4, \infty)$.**
2	Determine where $f(x) = -x^2 + 14x$ is increasing and decreasing.	**Increasing on $(-\infty, 7)$; decreasing on $(7, \infty)$.**
3	Determine where $f(x) = x^3 - 27x$ is increasing and decreasing.	**Increasing on $(-\infty, -3)$ and $(3, \infty)$; decreasing on $(-3, 3)$.**
4	Determine where $f(x) = x^3 + 6x^2$ is increasing and decreasing.	**Increasing on $(-\infty, -4)$ and $(0, \infty)$; decreasing on $(-4, 0)$.**
5	A differentiable function has $f'(x) = (x - 1)(x + 9)$. Determine where f is increasing and decreasing.	**Increasing on $(-\infty, -9)$ and $(1, \infty)$; decreasing on $(-9, 1)$.**
6	A differentiable function has $f'(x) = -(x + 6)(x - 8)$. Determine where f is increasing and decreasing.	**Decreasing on $(-\infty, -6)$ and $(8, \infty)$; increasing on $(-6, 8)$.**

PART 2 · APPLY IT

1. **Profit is increasing for $0 < x < 9$ and decreasing for $x > 9$.** $P'(x) = -2x + 18$, which is zero at $x = 9$. The derivative is positive before 9 and negative after 9 in the given domain.
2. **The viewer count is increasing for $0 < x < 8$ and decreasing for $8 < x < 18$.** $V'(x) = -2x + 16$, which is zero at $x = 8$. The derivative is positive from 0 to 8 and negative from 8 to 18.

CHALLENGE

$f'(x) = x^2(x^2 - 3)/(x^2 - 1)^2$. The function is increasing on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$. It is decreasing on $(-\sqrt{3}, -1)$, $(-1, 1)$, and $(1, \sqrt{3})$. The values $x = -1$ and $x = 1$ split the intervals because f is undefined there.

Accept equivalent interval notation and sign-chart reasoning. For the challenge, accept the decreasing interval $(-1, 1)$ split at $x = 0$ into $(-1, 0)$ and $(0, 1)$, since the derivative is zero but does not change sign at $x = 0$.