LESSON MAKER PRO

Peak Point

Use derivatives to locate maximums and minimums.

HIGH SCHOOL · MA.912.C.3

MA.912.C.3 "Apply derivatives to solve problems."

NAME _____ DATE _____

OPTIMIZE WITH DERIVATIVES

Critical numbers are inputs in the domain where f′ is zero or undefined. Use a sign change or a conclusive second-derivative test to classify local extrema. For a continuous function on a closed interval, compare values at all interior critical numbers and endpoints to find absolute extrema. A zero second derivative is inconclusive.

WORKED EXAMPLE

Find the maximum value of $p(x)=-3x^2+18x+5$.

1. Differentiate: $p'(x)=-6x+18$.
2. Set $p'(x)=0$: $-6x+18=0$, so $x=3$.
3. Since $p''(x)=-6<0$, this point gives a maximum.
4. Evaluate: $p(3)=-3(3^2)+18(3)+5=32$.

Answer: The maximum value is 32 at x=3.

PRACTICE 6 problems

1 For $f(x)=-2x^2+16x+1$, find the x-value and maximum value.

ANSWER _____

2 For $g(x)=x^2-14x+2$, find the x-value and minimum value.

ANSWER _____

3 For $h(x)=-x^2+14x-9$, find the x-value and maximum value.

ANSWER _____

4 For $k(x)=2x^2-16x+9$, find the x-value and minimum value.

ANSWER _____

5 A function has derivative $m'(x)=10x-40$ and has one critical point. Is the critical point a maximum or a minimum, and where is it?

ANSWER _____

6 For $n(x)=-7x^2+28x+2$, find the x-value and maximum value.

ANSWER _____

APPLY IT Show your work

1 A student council tracks the daily viewing rate, in thousands of views per day, for a short video t days after it is posted. The model is $V(t)=-t^2+14t+1$ for $0<=t<=14$. When is the viewing rate greatest, and what is its greatest value?

WORK SPACE

ANSWER

2 A student runs a game-night snack table. Its profit P(x), in dollars, is $P(x)=-x^2+20x-4$, where x is the price, in dollars, of a snack and $0<=x<=20$. What price gives the greatest profit, and what is that profit?

WORK SPACE

ANSWER

CHALLENGE One step up

On $0<=x<=8$, find the absolute maximum value of $q(x)=-4x^3+24x^2$ and the x-value where it occurs. Show derivative reasoning.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x)=-2x^2+16x+1$, find the x-value and maximum value.	**Maximum at x=4; f(4)=33.**
2	For $g(x)=x^2-14x+2$, find the x-value and minimum value.	**Minimum at x=7; g(7)=-47.**
3	For $h(x)=-x^2+14x-9$, find the x-value and maximum value.	**Maximum at x=7; h(7)=40.**
4	For $k(x)=2x^2-16x+9$, find the x-value and minimum value.	**Minimum at x=4; k(4)=-23.**
5	A function has derivative $m'(x)=10x-40$ and has one critical point. Is the critical point a maximum or a minimum, and where is it?	**It is a minimum at x=4.**
6	For $n(x)=-7x^2+28x+2$, find the x-value and maximum value.	**Maximum at x=2; n(2)=30.**

PART 2 · APPLY IT

1. **The daily viewing rate is greatest at t = 7 days: 50 thousand views per day.** $V'(t)=-2t+14=0$ gives $t=7$. Then $V(7)=50$, and the critical point is a maximum because $V''(t)=-2<0$.
2. **A price of $10 gives the greatest profit, which is $96.** $P'(x)=-2x+20=0$ gives $x=10$. Then $P(10)=96$, and $P''(x)=-2<0$ confirms a maximum.

CHALLENGE

$q'(x)=-12x^2+48x=-12x(x-4)$, so check x=0, x=4, and x=8. q(0)=0, q(4)=128, and q(8)=-512, so the absolute maximum is 128 at x=4.

Accept equivalent derivative forms. For the video model, require a daily viewing rate (50,000 views per day), not a cumulative total.