LESSON MAKER PRO

Rectangle Sum Lab

Estimate accumulated change with equal-width rectangles.

HIGH SCHOOL · MA.912.C.4.1

MA.912.C.4.1 "Interpret a definite integral as a limit of Riemann sums. Calculate the values of Riemann sums over equal subdivisions using left, right and midpoint evaluation points."

NAME _____ DATE _____

BUILD A RIEMANN SUM

Divide the interval [a,b] into n equal parts, where Delta x = (b-a)/n. For L_n use each left endpoint, for R_n use each right endpoint, and for M_n use each midpoint, then add the rectangle areas f(x) Delta x. As n increases, these sums approach the definite integral. A definite integral is the limit of Riemann sums as the maximum subinterval width tends to zero. Finite sums approximate the accumulated signed area; the limiting process defines it.

WORKED EXAMPLE

Approximate the integral from 11 to 19 of x dx using the midpoint sum M_8.

1. Delta x = (19-11)/8 = 1.
2. The midpoints are 23/2, 25/2, 27/2, 29/2, 31/2, 33/2, 35/2, and 37/2.
3. M_8 = 1(23/2 + 25/2 + 27/2 + 29/2 + 31/2 + 33/2 + 35/2 + 37/2).
4. M_8 = 120.

Answer: M_8 = 120

PRACTICE 6 problems

1 Find L_4 for f(x) = x on [0,12].

ANSWER _____

2 Find R_3 for f(x) = x + 1 on [0,6].

ANSWER _____

3 Find M_2 for f(x) = x^2 on [0,4].

ANSWER _____

4 Find L_3 for f(x) = 2x - 1 on [1,4].

ANSWER _____

5 Find R_4 for f(x) = 6 - x on [0,4].

ANSWER _____

6 Find M_5 for f(x) = 2x + 3 on [0,10].

ANSWER _____

APPLY IT Show your work

1 During a four-hour gaming charity stream, the donation rate is d(t) = 10t + 20 dollars per hour, where t is hours after the stream begins. Use M_4 to estimate the total donations during the stream.

2 During the first 3 minutes of track warm-up, a student's calorie-burning rate is c(t) = 4t + 10 calories per minute, where t is in minutes. Use L_3 to estimate the calories burned. For the simpler function f(x)=x on [0,1], show that a right sum with n equal pieces is R_n=(n+1)/(2n), using 1+…+n=n(n+1)/2. Find its limit and explain what happens to the rectangle widths.

CHALLENGE One step up

For f(x) = 12 - 2x on [0,4], use four equal subintervals. Find L_4 and R_4. Then give bounds for the integral from 0 to 4 of (12 - 2x) dx and explain why the bounds work.

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find L_4 for f(x) = x on [0,12].	**$L_4 = 54$**
2	Find R_3 for f(x) = x + 1 on [0,6].	**$R_3 = 30$**
3	Find M_2 for f(x) = x^2 on [0,4].	**$M_2 = 20$**
4	Find L_3 for f(x) = 2x - 1 on [1,4].	**$L_3 = 9$**
5	Find R_4 for f(x) = 6 - x on [0,4].	**$R_4 = 14$**
6	Find M_5 for f(x) = 2x + 3 on [0,10].	**$M_5 = 130$**

PART 2 · APPLY IT

1. **$160** Each subinterval has width 1 hour, with midpoints 1/2, 3/2, 5/2, and 7/2. Add the rates 25, 35, 45, and 55, then multiply by 1 hour.
2. **42 calories For f(x)=x, Δx=1/n and right heights i/n, so R_n=Σ(i/n)(1/n)=(n+1)/(2n). As n→∞, Δx→0 and R_n→1/2=$\int_0^1 x\,dx$.** Each subinterval has width 1 minute, and the left endpoints are 0, 1, and 2. Add c(0), c(1), and c(2). Require the factor Δx and the limit, not just an antiderivative or a single numerical approximation.

CHALLENGE

$L_4 = 36$ and $R_4 = 28$. Therefore, 28 <= integral from 0 to 4 of (12 - 2x) dx <= 36. Because f(x) is decreasing, right-endpoint rectangles underestimate and left-endpoint rectangles overestimate.

Accept equivalent arithmetic expressions and correctly labeled sums. For the challenge, accept any clear explanation that a decreasing function makes left sums overestimates and right sums underestimates.