LESSON MAKER PRO

One Branch Only

Restrict a domain to create an inverse function.

HIGH SCHOOL · MA.912.F.3.8

MA.912.F.3.8 "Produce an invertible function from a non-invertible function by restricting the domain."

NAME _____ DATE _____

CHOOSE ONE BRANCH

A function is invertible when each output comes from only one allowed input. For a U-shaped or upside-down U-shaped graph, restrict the domain to one side of the vertex. Then solve y = f(x), using the sign that matches the branch you chose.

WORKED EXAMPLE

Make $h(x) = (x - 13)^2 + 17$ invertible by using the branch $x >= 13$. Then find $h^{-1}(x)$.

1. Set $y = (x - 13)^2 + 17$.
2. Subtract 17: $y - 17 = (x - 13)^2$.
3. Because $x >= 13$, use $x - 13 = \sqrt{y - 17}$.
4. Solve for x, then switch variables: $h^{-1}(x) = 13 + \sqrt{x - 17}$.

Answer: Restricted domain: $x >= 13$. $h^{-1}(x) = 13 + \sqrt{x - 17}$.

PRACTICE 6 problems

1 For $f(x) = x^2$, restrict the domain to $x >= 0$. Write $f^{-1}(x)$.

ANSWER _____

2 For $g(x) = (x - 4)^2 - 2$, restrict the domain to $x <= 4$. Write $g^{-1}(x)$.

ANSWER _____

3 For $p(x) = |x - 5| + 1$, restrict the domain to $x >= 5$. Write $p^{-1}(x)$.

ANSWER _____

4 For $q(x) = x^2 + 6x$, restrict the domain to $x >= -3$. Write $q^{-1}(x)$.

ANSWER _____

5 For $r(x) = -(x - 2)^2 + 8$, restrict the domain to $x <= 2$. Write $r^{-1}(x)$.

ANSWER _____

6 For $t(x) = (x - 9)^2$, restrict the domain to $x >= 9$. In one sentence, explain why this restriction makes t invertible.

ANSWER _____

APPLY IT Show your work

1 A student records a basketball's height h, in feet, t seconds after release: $h(t) = -2(t - 3)^2 + 18$, where $0 <= t <= 6$. A video tool knows the height and needs the time on the downward path. State a domain restriction for h and write the inverse function, including the inverse's domain.

WORK SPACE

ANSWER

2 A club's profit P, in dollars, from selling q items is $P(q) = -(q - 8)^2 + 64$, where $0 <= q <= 16$. The treasurer knows the profit and wants the larger possible number of items sold. State a domain restriction for P and write the inverse function, including the inverse's domain. Here q must be a whole number.

WORK SPACE

ANSWER

CHALLENGE One step up

Let $F(x) = x^2 - 10x + 21$. Use the full interval on which F is monotonic and that contains $x = 9$. State the restricted domain, write $F^{-1}(x)$, state the range of the restricted function, and explain why the other square-root branch is not used.

WORK SPACE AND ANSWER

ANSWER KEY

One Branch Only

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x^2$, restrict the domain to $x >= 0$. Write $f^{-1}(x)$.	**$f^{-1}(x) = \text{sqrt}(x)$**
2	For $g(x) = (x - 4)^2 - 2$, restrict the domain to $x <= 4$. Write $g^{-1}(x)$.	**$g^{-1}(x) = 4 - \text{sqrt}(x + 2)$**
3	For $p(x) = \lvert x - 5\rvert + 1$, restrict the domain to $x >= 5$. Write $p^{-1}(x)$.	**$p^{-1}(x) = x + 4$**
4	For $q(x) = x^2 + 6x$, restrict the domain to $x >= -3$. Write $q^{-1}(x)$.	**$q^{-1}(x) = -3 + \text{sqrt}(x + 9)$**
5	For $r(x) = -(x - 2)^2 + 8$, restrict the domain to $x <= 2$. Write $r^{-1}(x)$.	**$r^{-1}(x) = 2 - \text{sqrt}(8 - x)$**
6	For $t(x) = (x - 9)^2$, restrict the domain to $x >= 9$. In one sentence, explain why this restriction makes t invertible.	**On $x >= 9$, t is increasing, so each output has only one allowed input.**

PART 2 · APPLY IT

1. **Restrict h to $3 <= t <= 6$. $h^{-1}(x) = 3 + \text{sqrt}((18 - x)/2)$, with $0 <= x <= 18$.** Use the descending branch, which begins at the vertex when $t = 3$. Solve $x = -2(t - 3)^2 + 18$ and choose the positive square-root branch because $t - 3 >= 0$.
2. **Restrict q to the whole numbers 8 through 16. $P^{-1}(x) = 8 + \sqrt{}(64 - x)$, with inverse domain {0, 15, 28, 39, 48, 55, 60, 63, 64} dollars.** Use the branch after the maximum profit, where $q >= 8$. Solve $x = -(q - 8)^2 + 64$ and choose the positive square-root branch because $q - 8 >= 0$. The possible profits on this integer branch are P(8), …, P(16) = 64, 63, 60, 55, 48, 39, 28, 15, 0; only those values belong to the inverse domain.

CHALLENGE

Restricted domain: $x >= 5$. $F^{-1}(x) = 5 + \text{sqrt}(x + 4)$. Range: $x >= -4$. The other branch gives inputs $x <= 5$, so it does not match the chosen domain.

Accept equivalent radical forms, such as $3 + \text{sqrt}((18 - x)/2)$ or $3 + \text{sqrt}(18 - x)/\text{sqrt}(2)$. For inverse functions, accept an explicitly stated inverse domain or the equivalent range of the restricted original function.