LESSON MAKER PRO

Vote Method Matchup

Use Borda count to compare ranked-choice results.

HIGH SCHOOL · MA.912.LT.3.2

MA.912.LT.3.2 "Analyze election data using election theory techniques. Explain how Arrow's Impossibility Theorem may be related to the fairness of the outcome of the election."

NAME _____ DATE _____

BORDA COUNT

For three choices, Borda count gives 2 points for a first choice, 1 point for a second choice, and 0 points for a last choice. Multiply each point value by the number of voters, then add each candidate's points. Different voting methods can produce different winners. Arrow's theorem concerns rules that turn individual rankings of at least three alternatives into a complete, consistent social ranking. No such rule can satisfy all four conditions together: allow every possible preference profile; respect unanimous preferences; make each pair's social order depend only on voters' rankings of that pair; and avoid a dictator whose preferences always decide.

WORKED EXAMPLE

Use Borda count: 16 voters rank A > B > C, 9 rank B > C > A, and 7 rank C > A > B.

1. A: 2(16) + 0(9) + 1(7) = 39.
2. B: 1(16) + 2(9) + 0(7) = 34.
3. C: 0(16) + 1(9) + 2(7) = 23, so A has the most points.

Answer: A wins with 39 points.

PRACTICE 5 problems

1 Use Borda count: 12 rank A > B > C, 5 rank B > C > A, and 4 rank C > A > B.

ANSWER _____

2 Use Borda count: 10 rank D > E > F, 6 rank E > F > D, and 3 rank F > D > E.

ANSWER _____

3 For four choices, use 3, 2, 1, 0 points. 8 rank G > H > I > J, 5 rank H > I > J > G, and 2 rank I > J > G > H.

ANSWER _____

4 Use Borda count: 11 rank A > B > C, 8 rank B > C > A, and 6 rank C > B > A.

ANSWER _____

5 Use Borda count: 13 rank X > Y > Z, 4 rank Y > Z > X, and 2 rank Z > X > Y.

ANSWER _____

APPLY IT Show your work

1 The media club is choosing a movie-night activity. 13 students rank Game Night > Bake Sale > Movie, 10 rank Bake Sale > Movie > Game Night, and 4 rank Movie > Game Night > Bake Sale. Use Borda count.

WORK SPACE

ANSWER

2 Students are choosing a concert finale. For four choices, use 3, 2, 1, 0 points. 15 rank R > S > T > U, 8 rank S > T > U > R, and 3 rank T > U > S > R.

WORK SPACE

ANSWER

CHALLENGE One step up

18 voters rank A > B > C, 17 rank B > C > A, and 15 rank C > B > A. Find the plurality winner and the Borda winner. Then explain what the different results show about fairness and Arrow's Impossibility Theorem.

WORK SPACE AND ANSWER

ANSWER KEY

Vote Method Matchup

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use Borda count: 12 rank A > B > C, 5 rank B > C > A, and 4 rank C > A > B.	**A wins with 28 points.**
2	Use Borda count: 10 rank D > E > F, 6 rank E > F > D, and 3 rank F > D > E.	**D: 23, E: 22, F: 12. D wins.**
3	For four choices, use 3, 2, 1, 0 points. 8 rank G > H > I > J, 5 rank H > I > J > G, and 2 rank I > J > G > H.	**G: 26, H: 31, I: 24, J: 9. H wins.**
4	Use Borda count: 11 rank A > B > C, 8 rank B > C > A, and 6 rank C > B > A.	**B wins with 33 points.**
5	Use Borda count: 13 rank X > Y > Z, 4 rank Y > Z > X, and 2 rank Z > X > Y.	**X: 28, Y: 21, Z: 8. X wins.**

PART 2 · APPLY IT

1. **Bake Sale wins with 33 points.** Game Night earns 30 points, Bake Sale earns 33 points, and Movie earns 18 points.
2. **S wins with 57 points.** The totals are R: 45, S: 57, T: 40, and U: 14.

CHALLENGE

Plurality winner: A. Borda winner: B; totals A: 36, B: 67, C: 47. The different winners show that the chosen method matters. This example does not prove Arrow's theorem or show that all notions of fairness are impossible.

Accept correct Borda totals with equivalent working. The challenge must distinguish method-dependent winners from the theorem's specific assumptions and joint conditions; do not accept "no system satisfies any fairness rule." Arrow's theorem concerns rules that turn individual rankings of at least three alternatives into a complete, consistent social ranking. No such rule can satisfy all four conditions together: allow every possible preference profile; respect unanimous preferences; make each pair's social order depend only on voters' rankings of that pair; and avoid a dictator whose preferences always decide.