Lines Tell Rates

Connect slopes, equations, and proportional relationships.

CCSS.Math.Content.8.EE.B · Understand the connections between proportional relationships, lines, and linear equations.

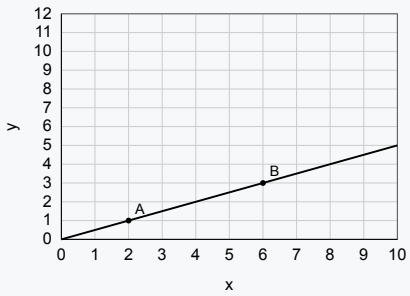
Name _____ Date _____

The Big Idea

A proportional relationship has equation y = mx: its line passes through (0, 0), and m is both unit rate and slope. On a nonvertical line, slope is rise/run. Right triangles along the same line have the same rise/run ratio. In y = mx + b, b is the y-intercept; a nonzero b means the relationship is not proportional.

Worked example

The graph shows y = x/2. Read (2, 1) and (6, 3). What is the slope, and why is the relationship proportional?



1. From A to B: rise = 3 - 1 = 2; run = 6 - 2 = 4. Slope = 2/4 = 1/2.
2. The line goes through (0, 0), so y is always half of x.

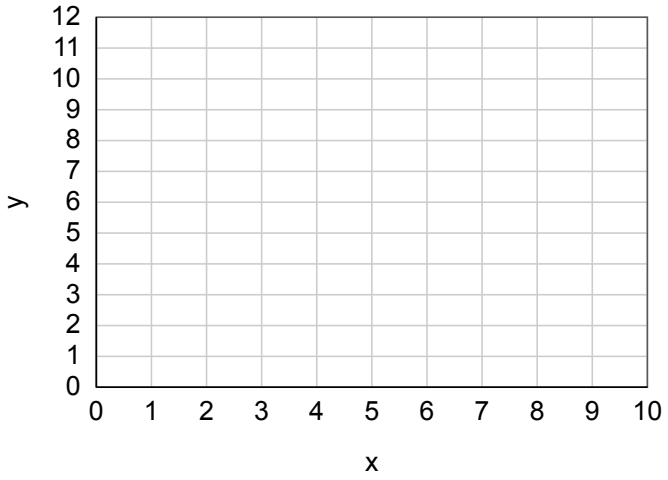
Slope and unit rate are 1/2. The origin lies on the line.

1. Practice

Graph y = 3x on this grid. Plot (0, 0), (1, 3), (2, 6) and (3, 9), then draw the line. State the unit rate.

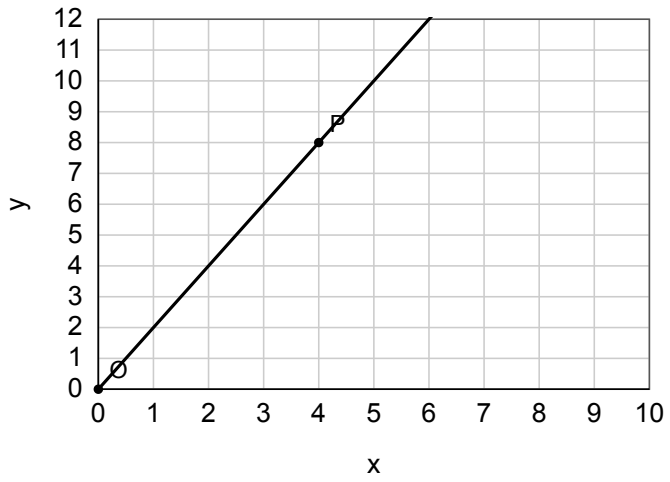
_____

_____



2. Practice

The graph shows a second relationship. Find its slope and equation from the labeled points. Is it proportional?

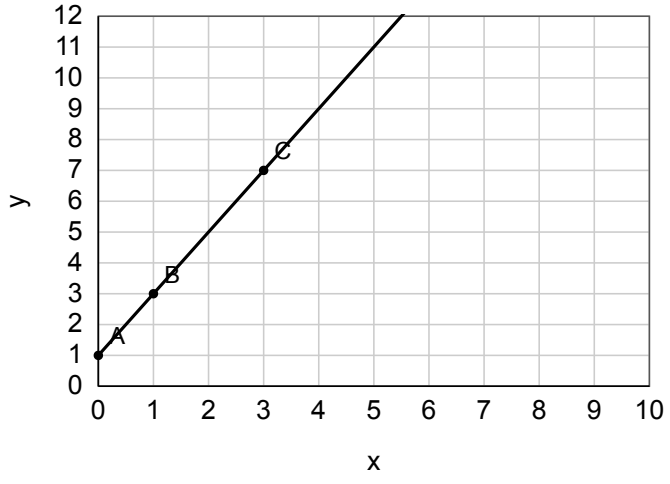


3. Practice

Compare Practice 1 and 2 on their matching scales. Which relationship has the greater unit rate? Explain how the graphs show it.

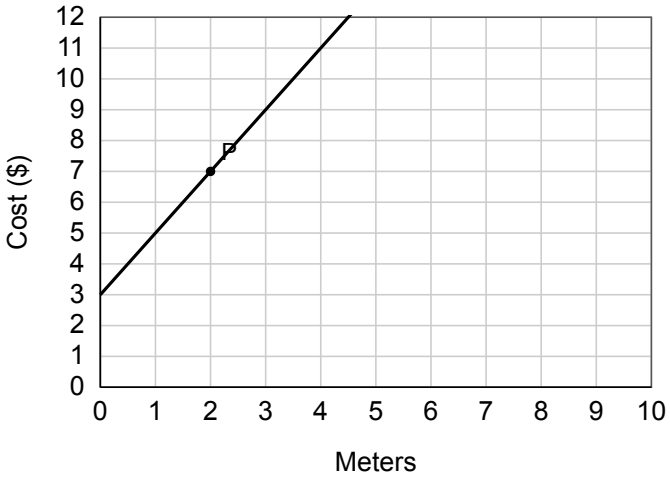
4. Practice

Draw two slope triangles along this line: from A(0, 1) to B(1, 3), and from A to C(3, 7). Use horizontal runs and vertical rises. Compare the ratios.



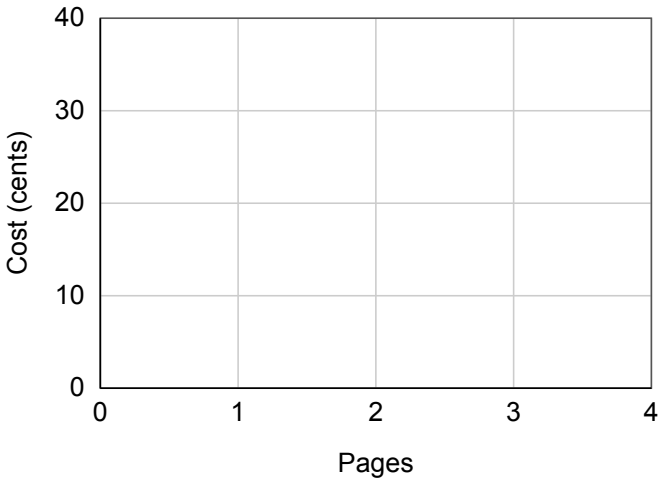
5. Apply

A fictional craft service charges $3 setup plus $2 per meter of ribbon. The graph models y = 2x + 3. Interpret (2, 7), the slope and the y-intercept. Is the cost proportional to length?



6. Apply

A fictional copying service charges $0.10 per page with no fee. Write its equation with y in cents. Plot costs for 0, 1, 2 and 3 pages; use the labeled scale. What does (3, 30) mean?



7. Challenge

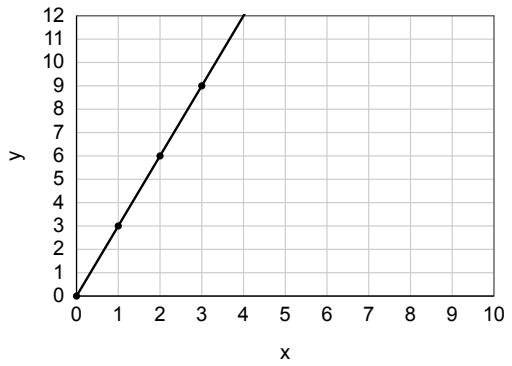
A line crosses the y-axis at (0, 3) and has slope 2. Use rise/run to explain why ANY point (x, y) on it satisfies y = 2x + 3. Include the intercept case x = 0.

What to accept

Check full plotted lines for abstract relationships and the specified separate points for whole-page copying costs. Scaled copying grid labels are cents, not grid-row numbers. For slope triangles, accept horizontal-then-vertical steps A to (1,1) to B and A to (3,1) to C; their rise/run ratios match. The challenge uses directed changes for negative x; avoid dividing by x at the intercept.

1. Practice

A straight line through all four points; unit rate 3.



2. Practice

Slope = (8 - 0)/(4 - 0) = 2; y = 2x. Yes, its line goes through the origin.

3. Practice

y = 3x has the greater rate: 3 > 2. For the same positive x, it has the larger y and the steeper line.

4. Practice

Small: rise/run = 2/1. Large: 6/3. Both equal 2; the triangles are similar right triangles along the same line.

5. Apply

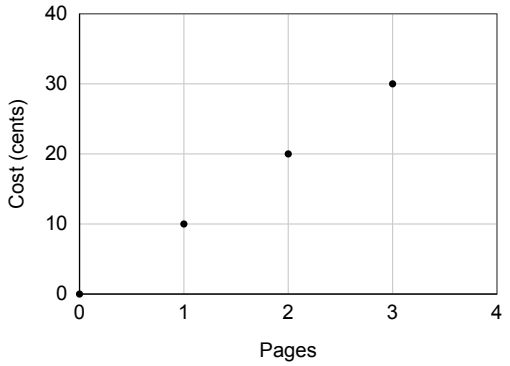
$7 for 2 meters; slope $2 per meter; intercept $3 setup. Not proportional.

The graph crosses the y-axis at 3, not 0. At x = 2, y = 2(2) + 3 = 7.

6. Apply

y = 10x cents. Three pages cost 30 cents.

Points are (0, 0), (1, 10), (2, 20), (3, 30). The origin and unit rate show proportional cost.



7. Challenge

From (0, 3), horizontal change is x and vertical change is y - 3. Slope 2 gives y - 3 = 2x, so y = 2x + 3. At x = 0, y = 3, which also satisfies the equation.