LESSON MAKER PRO

Patterns That Grow

Compare equal differences and equal factors.

HIGH SCHOOL · HSF-LE.A.1a

CCSS.Math.Content.HSF-LE.A.1a "Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals."

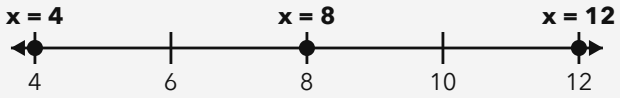
NAME _____ DATE _____

THE BIG IDEA

When x changes by equal amounts, a linear function changes by the same amount each time. An exponential function is multiplied by the same factor each time, so compare ratios instead of differences.

WORKED EXAMPLE

Assume f is either linear or exponential. For f(4)=19, f(8)=31 and f(12)=43, decide which model fits.



1. 8 - 4 = 4 and 12 - 8 = 4.
2. 31 - 19 = 12 and 43 - 31 = 12.
3. Among these two model families, equal differences and unequal ratios identify the linear model.

Answer: Linear; the output increases by 12 for every increase of 4 in x.

PRACTICE 4 problems

1 Assume the function is linear or exponential. For f(1) = 9, f(3) = 16, and f(5) = 23, classify the function. State the output change.

ANSWER _____

2 Assume the function is linear or exponential. At x = 0, 3, and 6, a function has values 5, 25, and 125. Classify the function. State the factor.

ANSWER _____

3 A linear function has f(0) = 10 and f(5) = 30. Find f(10).

ANSWER _____

4 Assume the function is linear or exponential. Over equal x-intervals, the outputs are 81, 72, 63, and 54. Is the function linear or exponential? State the pattern.

ANSWER _____

_____

APPLY IT Show your work

1 Assume the view counts follow a linear or exponential model. The table shows views on a student's music channel. Is the growth linear or exponential? Predict the views after 5 hours.

MUSIC CHANNEL VIEWS

Hours after post	Views
0	120
1	150
2	180
3	210

ANSWER _____

2 A study-account post has 7 shares at hour 0. Its shares triple each hour. What model is this, and how many shares will it have after 5 hours?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For x=0,1,2,3, list outputs of $f(x)=9x+7$ and $g(x)=5\times3^x$. Then prove the equal-interval properties generally. Let h be any fixed positive step. Simplify $F(x+h)-F(x)$ for $F(x)=mx+b$, and $G(x+h)/G(x)$ for $G(x)=a\times c^x$, where $a\neq0$ and $c>0$, $c\neq1$. Explain why each result depends on h but not the starting x. Apply the results to f and g. Use extra paper for your derivation.

WORK SPACE AND ANSWER

Patterns That Grow

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Assume the function is linear or exponential. For f(1) = 9, f(3) = 16, and f(5) = 23, classify the function. State the output change.	**Linear; the output increases by 7 for every increase of 2 in x.**
2	Assume the function is linear or exponential. At x = 0, 3, and 6, a function has values 5, 25, and 125. Classify the function. State the factor.	**Exponential; the output is multiplied by 5 each interval.**
3	A linear function has f(0) = 10 and f(5) = 30. Find f(10).	**50**
4	Assume the function is linear or exponential. Over equal x-intervals, the outputs are 81, 72, 63, and 54. Is the function linear or exponential? State the pattern.	**Linear; subtract 9 each interval.**

PART 2 · APPLY IT

1. **Linear; 270 views.** The views increase by 30 each hour. Continue the equal difference for two more hours: 210 + 30 + 30 = 270.
2. **Exponential; 1,701 shares.** The equal factor is 3. Compute $7 \times 3^5 = 7 \times 243 = 1{,}701$.

CHALLENGE

f outputs: 7,16,25,34; g outputs: 5,15,45,135. F(x+h)-F(x)=[m(x+h)+b]-(mx+b)=mh. G(x+h)/G(x)=$[a \times c^{x+h}]/[a \times c^x]=c^h$. Since a≠0 and c>0, the denominator is nonzero. For fixed h both results are independent of starting x, establishing equal differences or equal factors over equal intervals. Thus f(x+h)-f(x)=9h and g(x+h)/g(x)=3^h; h=1 gives difference9 and factor3.

Require both original output lists and a general equal-step derivation or equivalent valid proof, with independence from starting x explained. Finite samples alone do not prove behavior at every x. Classification tasks explicitly assume the linear/exponential model families; accept correct matching differences/ratios. For a decreasing linear function, a negative constant difference is valid.