LESSON MAKER PRO

Parallelogram Power Moves

Use opposite parts and diagonals to solve geometry problems.

HIGH SCHOOL · HSG-CO.C.11

CCSS.Math.Content.HSG-CO.C.11 "Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals."

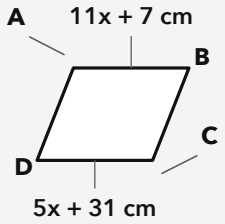
NAME _____ DATE _____

PARALLELOGRAM FACTS

In a parallelogram, opposite sides and opposite angles are congruent, and diagonals bisect each other. A rectangle also has congruent diagonals; since it is a parallelogram, those diagonals bisect each other.

WORKED EXAMPLE

Use the figure. Find x and the length of AB.

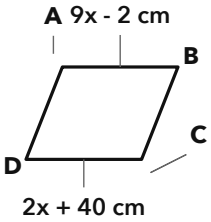


1. Opposite sides of a parallelogram are congruent, so 11x + 7 = 5x + 31.
2. Subtract 5x: 6x + 7 = 31.
3. Subtract 7 and divide by 6: x = 4.
4. AB = 11(4) + 7 = 51 cm.

Answer: x = 4; AB = 51 cm

PRACTICE 6 problems

1 Use the figure. Find x and the length of AB.



ANSWER _____

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2 In parallelogram PQRS, m∠P = 116°. Find m∠R and m∠Q.

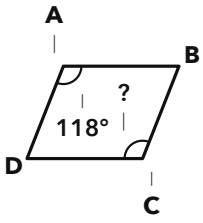
ANSWER _____

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3 Diagonals AC and BD of parallelogram ABCD intersect at M. AM = 10x + 3 and MC = 13x - 21. Find x.

ANSWER _____

4 Use the figure. Find m∠C and m∠B.



ANSWER _____

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5 The diagonals of parallelogram EFGH intersect at N. If EG = 46 cm, find NG.

ANSWER _____

6 Rectangle WXYZ has diagonal WY = 34 cm. Find the length of diagonal XZ.

ANSWER _____

APPLY IT Show your work

1 The student council designs a parallelogram-shaped frame for a festival photo booth. On the blueprint, one side is 14x - 18 cm and the opposite side is 8x + 24 cm. Find x and the length of each of those sides.

WORK SPACE

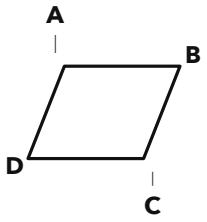
ANSWER _____

2 The two diagonal braces of a rectangular esports display cross at M. One diagonal is 74 cm long. Find the length of the other diagonal and the distance from M to either endpoint on that diagonal.

WORK SPACE

ANSWER _____

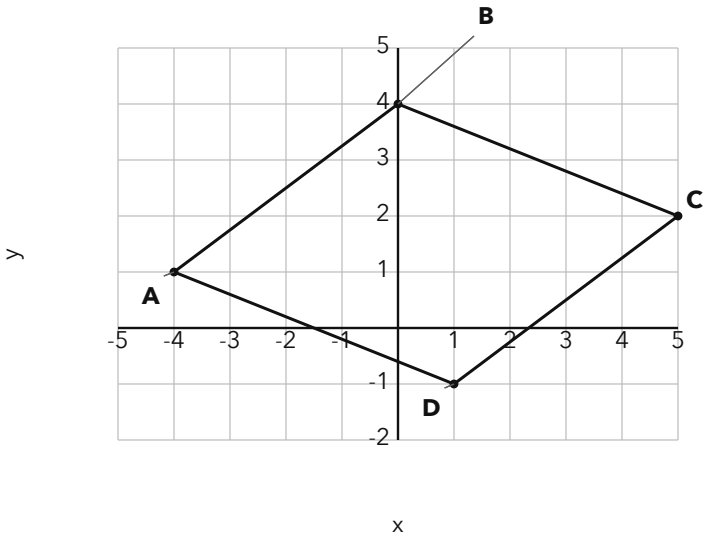
3 Prove a theorem for ANY parallelogram ABCD, with vertices in order. Draw diagonal AC on the sketch. Given AB parallel CD and BC parallel AD, identify two pairs of equal alternate interior angles. Use the common side AC to prove triangle ABC congruent to triangle CDA by ASA, then conclude both pairs of opposite sides are congruent. Write a reason for each step; measuring this sketch alone is not a proof. Use extra paper if needed.



ANSWER _____

CHALLENGE One step up

Use the coordinate grid. Explain how the slopes of opposite sides and the midpoints of the diagonals show that ABCD is a parallelogram whose diagonals bisect each other.



WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use the figure. Find x and the length of AB.	**x = 6; AB = 52 cm**
2	In parallelogram PQRS, m∠P = 116°. Find m∠R and m∠Q.	**m∠R = 116°; m∠Q = 64°**
3	Diagonals AC and BD of parallelogram ABCD intersect at M. AM = 10x + 3 and MC = 13x - 21. Find x.	**x = 8**
4	Use the figure. Find m∠C and m∠B.	**m∠C = 118°; m∠B = 62°**
5	The diagonals of parallelogram EFGH intersect at N. If EG = 46 cm, find NG.	**NG = 23 cm**
6	Rectangle WXYZ has diagonal WY = 34 cm. Find the length of diagonal XZ.	**XZ = 34 cm**

PART 2 · APPLY IT

1. **x = 7; each side is 80 cm** Set opposite side lengths equal: 14x - 18 = 8x + 24. Solve x = 7, then substitute to get 80 cm.
2. **Other diagonal = 74 cm; distance from M to either endpoint = 37 cm** Rectangle diagonals are congruent, so the other diagonal is 74 cm. Diagonals of a parallelogram bisect each other, so M divides it into two 37 cm segments.
3. **AB=CD and BC=DA for every parallelogram ABCD.** Because AB is parallel CD, angle BAC equals angle DCA. Because BC is parallel AD, angle BCA equals angle DAC. AC equals CA by reflexivity. Thus triangle ABC is congruent to triangle CDA by ASA. Corresponding sides give AB=CD and BC=DA. The argument uses only parallelism and a common diagonal, so it holds for any parallelogram, not merely this sketch.

CHALLENGE

AB and DC have slope 3/4, and BC and AD have slope -2/5. The midpoint of AC is (1/2, 3/2), and the midpoint of BD is (1/2, 3/2), so opposite sides are parallel and the diagonals bisect each other.

Accept correct equivalent algebraic work and units where shown. For the challenge, students must identify both pairs of equal slopes and the shared diagonal midpoint. For Apply 3, require a connected general proof with both parallel-line angle reasons, common AC, ASA and corresponding-side conclusions. Accept an equivalent valid general proof; particular measurements alone are insufficient.