LESSON MAKER PRO

Circle Intersection Plans

Use SSS construction logic to locate a triangle's third vertex.

HIGH SCHOOL · MA.912.GR.5

MA.912.GR.5 "Make formal geometric constructions with a variety of tools and methods."

NAME _____ DATE _____

BUILD WITH TWO CIRCLES

Draw a base, then compass circles centered at its endpoints using the other two side lengths. Two intersections give two mirror-image nondegenerate triangles. One tangent intersection gives a collinear point, not a valid triangle; no intersection gives no third-vertex location. A valid triangle requires the largest side to be less than the sum of the other two.

WORKED EXAMPLE

Plan an SSS construction for triangle PQR with PQ = 7 cm, PR = 6 cm, and QR = 5 cm.

1. Draw segment PQ with length 7 cm.
2. Draw a circle centered at P with radius 6 cm.
3. Draw a circle centered at Q with radius 5 cm.
4. Label either circle intersection R, then connect R to P and Q.

Answer: Two mirror-image triangles are possible.

PRACTICE 5 problems

1 Can a triangle with side lengths 3 cm, 8 cm, and 10 cm be constructed?

ANSWER _____

2 Can a triangle with side lengths 2 cm, 4 cm, and 9 cm be constructed?

ANSWER _____

3 Points A and B are 11 cm apart. Point C must be 8 cm from A and 10 cm from B. How many possible locations can C have?

ANSWER _____

4 Points A and B are 13 cm apart. Point C must be 3 cm from A and 10 cm from B. How many possible locations can C have?

ANSWER _____

5 Points A and B are 15 cm apart. Point C must be 4 cm from A and 9 cm from B. How many possible locations can C have?

ANSWER _____

APPLY IT Show your work

1 A robotics team needs a triangular brace with side lengths 9 cm, 12 cm, and 16 cm. Can the team construct the brace? State the circle construction plan. Then actually construct the brace on teacher-provided unlined paper using a ruler, compass and straightedge (or an accessible digital compass/straightedge tool). Label base endpoints P and Q and an intersection R. Keep both construction circles visible, connect the three sides, and check PQ = 16 cm, PR = 9 cm and QR = 12 cm. Submit the completed construction with your plan.

WORK SPACE

ANSWER _____

2 Two concert light poles are 18 m apart. A camera mount must be 11 m from one pole and 13 m from the other. How many possible mount locations are there?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Segment AB is 20 cm long. Point C must be 13 cm from B, and AC must be a positive integer less than 12 cm. List every possible length of AC that allows triangle ABC to be constructed.

WORK SPACE AND ANSWER

ANSWER KEY # Circle Intersection Plans

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Can a triangle with side lengths 3 cm, 8 cm, and 10 cm be constructed?	**Yes.**
2	Can a triangle with side lengths 2 cm, 4 cm, and 9 cm be constructed?	**No.**
3	Points A and B are 11 cm apart. Point C must be 8 cm from A and 10 cm from B. How many possible locations can C have?	**2 locations.**
4	Points A and B are 13 cm apart. Point C must be 3 cm from A and 10 cm from B. How many possible locations can C have?	**1 tangent location; no nondegenerate triangle.**
5	Points A and B are 15 cm apart. Point C must be 4 cm from A and 9 cm from B. How many possible locations can C have?	**0 locations.**

PART 2 · APPLY IT

1. **Yes; completed labeled SSS construction with PQ = 16 cm, PR = 9 cm, QR = 12 cm.** Draw a 16 cm base, then draw circles with radii 9 cm and 12 cm from the base endpoints. Because 9 + 12 is greater than 16, the circles intersect. An intersection is the third vertex of the brace. Teacher checks the actual constructed output: base PQ is 16 cm, circles centered at P/Q have radii 9/12 cm, R lies at an intersection, and the three connected sides match those lengths. Either reflected intersection is valid; a written plan alone is insufficient.
2. **2 locations.** Draw circles centered at the poles with radii 11 m and 13 m. The circles intersect twice, once on each side of the segment between the poles.

CHALLENGE

AC can be 8 cm, 9 cm, 10 cm, or 11 cm.

Accept equivalent construction language that names the base segment and the two correct circle radii. For location counts, accept "two," "one," or "none" in place of numerals. Supply unlined paper and safe construction tools, support their use, and inspect the actual completed brace. For Practice 4, preserve one point-location answer: 3 + 10 = 13 gives tangency and a collinear configuration, not a valid triangle.