

One Third Volumes

Compare matching solids by base and height.

GRADE 8 • 8.MG.2.c

8.MG.2.c "Examine and explain the relationship between the volume of cones and cylinders, and the volume of rectangular prisms and square based pyramids."

NAME _____

DATE _____

THE ONE-THIRD RULE

A cone has $\frac{1}{3}$ the volume of a cylinder when the two solids have the same base area and height. A square-based pyramid has $\frac{1}{3}$ the volume of a square prism with the same base area and height, so the prism or cylinder has 3 times the volume.

WORKED EXAMPLE

A square prism has a base side length of 11 cm and a height of 13 cm. A square-based pyramid has the same base and height. Find the volume of each solid.

1. Base area = $11 \times 11 = 121 \text{ cm}^2$.
2. Prism volume = $121 \times 13 = 1573 \text{ cm}^3$.
3. Pyramid volume = $1573/3 = 524 \frac{1}{3} \text{ cm}^3$.

**Answer: Square prism: 1573 cm^3 ;
square-based pyramid: $524 \frac{1}{3} \text{ cm}^3$.**

PRACTICE 6 problems

- 1** A cone and a cylinder each have a base area of 24 cm^2 and a height of 5 cm. Find the volume of each solid.

ANSWER _____

- 2** A cone has a volume of 36 m^3 . A cylinder has the same base area and height as the cone. What is the cylinder's volume?

ANSWER _____

- 3** A square prism has a base side length of 6 cm and a height of 4 cm. A square-based pyramid has the same base and height. Find the volume of each solid.

ANSWER _____

- 4** A square-based pyramid has a base side length of 8 cm and a height of 15 cm. What is its volume?

ANSWER _____

- 5** A cylinder has a radius of 2 cm and a height of 18 cm. A cone has the same radius and height. Find the volume of each solid in terms of pi.

ANSWER _____

- 6** A square-based pyramid has a volume of 70 cm^3 . A square prism has the same base and height. What is the prism's volume?

ANSWER _____

APPLY IT Show your work

- 1** At a robotics club fundraiser, a cylindrical prize container has a radius of 4 cm and a height of 12 cm. A cone-shaped prize container has the same radius and height. How much does each container hold, in terms of pi?

WORK SPACE

ANSWER _____

- 2** The art club makes a square-based pyramid display with a base side length of 10 cm and a height of 9 cm. A square storage crate has the same base and height. Find the volume of the display and the crate.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A square-based pyramid is 9 cm tall and has a volume of 192 cm^3 . A square prism has the same square base and height. What is the side length of the square base, and what is the prism's volume? Explain your reasoning.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A cone and a cylinder each have a base area of 24 cm^2 and a height of 5 cm. Find the volume of each solid.	Cylinder: 120 cm^3; cone: 40 cm^3.
2	A cone has a volume of 36 m^3 . A cylinder has the same base area and height as the cone. What is the cylinder's volume?	108 m^3
3	A square prism has a base side length of 6 cm and a height of 4 cm. A square-based pyramid has the same base and height. Find the volume of each solid.	Square prism: 144 cm^3; square-based pyramid: 48 cm^3.
4	A square-based pyramid has a base side length of 8 cm and a height of 15 cm. What is its volume?	320 cm^3
5	A cylinder has a radius of 2 cm and a height of 18 cm. A cone has the same radius and height. Find the volume of each solid in terms of pi.	Cylinder: $72\pi \text{ cm}^3$; cone: $24\pi \text{ cm}^3$.
6	A square-based pyramid has a volume of 70 cm^3 . A square prism has the same base and height. What is the prism's volume?	210 cm^3

PART 2 • APPLY IT

- Cylinder: $192\pi \text{ cm}^3$; cone: $64\pi \text{ cm}^3$.** The cylinder volume is $\pi \times 4^2 \times 12 = 192\pi \text{ cm}^3$. The matching cone has $\frac{1}{3}$ of that volume, or $64\pi \text{ cm}^3$.
- Square-based pyramid display: 300 cm^3 ; square storage crate: 900 cm^3 .** The square base area is $10 \times 10 = 100 \text{ cm}^2$, so the crate volume is $100 \times 9 = 900 \text{ cm}^3$. The pyramid display has $\frac{1}{3}$ of the crate's volume, or 300 cm^3 .

CHALLENGE

The base side length is 8 cm, and the prism's volume is 576 cm^3 . The prism has 3 times the pyramid's volume, so its volume is $3 \times 192 = 576 \text{ cm}^3$. Its base area is $576/9 = 64 \text{ cm}^2$, and a square with area 64 cm^2 has side length 8 cm.

Accept equivalent mixed-number or improper-fraction forms for $524 \frac{1}{3} \text{ cm}^3$. For answers involving pi, accept equivalent expressions such as $24 \times \pi \text{ cm}^3$.